

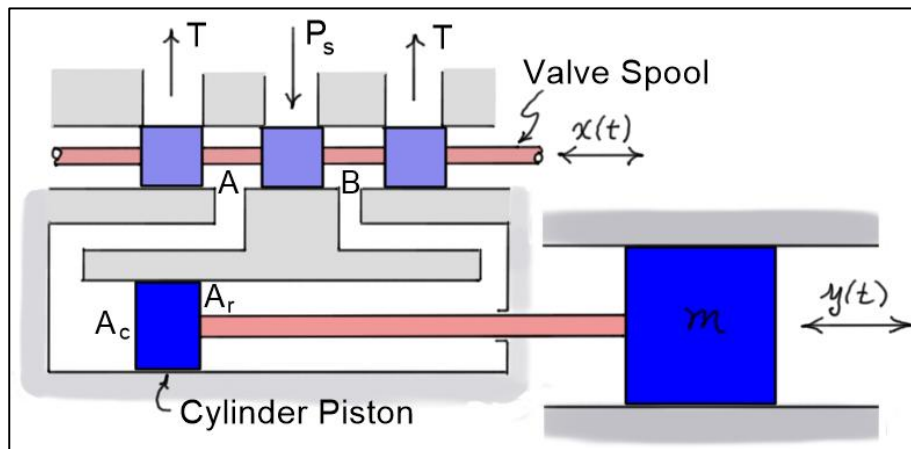
ME 4710 Motion and Control

Hydraulic Positioning System II

References: Dorf and Bishop, *Modern Control Systems*, 9th Ed., Prentice-Hall, 2001.
 Parker *Design Engineer's Handbook: Vol. 1 Hydraulics*, Bulletin 0292-B1-H, 2001.

Positioning System

- Incompressible fluid
- A_c = cap end piston area
- A_r = rod end piston area
- m = mass of load
- b = damping coefficient
- P_s = constant supply pressure
- P = pressure on the piston
- $p = \Delta P$, the change in P
- X = valve spool position
- $x = \Delta X$, the change in X
- Y = load position
- $y = \Delta Y$, the change in Y



Operation

- If $X > 0$, then the **pressure source** is **applied** to the **A port** of the valve and the **cap end** of the cylinder causing the load to **move** to the **right**. **Return flow** to the tank is through the **B port**.
- If $X < 0$, then the **pressure source** is **applied** to the **B port** of the valve and the **rod end** of the cylinder causing the load to **move** to the **left**. **Return flow** to the tank is through the **A port**.

Flow Model

If $X > 0$, then the pressure source is applied to the A port of the valve. As a result, fluid flows into the cap end of the cylinder and out of the rod end. The flow rate through the valve is a function of X the spool position and the pressures on either side of the piston.

$$Q_A = g_A(X, P_A) \quad \text{and} \quad Q_B = g_B(X, P_B) \quad (1)$$

To simplify the model, Eqs. (1) can be *linearized* about some *operational* (set) *points* (X_0, P_{A0}) and (X_0, P_{B0}) . This is done using a *Taylor series expansion*. The change in flow rates can be written as

$$\begin{aligned} q_A \triangleq \Delta Q_A &\approx \left(\frac{\partial g_A}{\partial X} \right)_{X_0, P_{A0}} \Delta X + \left(\frac{\partial g_A}{\partial P_A} \right)_{X_0, P_{A0}} \Delta P_A \\ &= (k_{xA})x - (k_{pA})p_A \end{aligned} \quad (2)$$

$$\begin{aligned} q_B \triangleq \Delta Q_B &\approx \left(\frac{\partial g_B}{\partial X} \right)_{X_0, P_{B0}} \Delta X + \left(\frac{\partial g_B}{\partial P_B} \right)_{X_0, P_{B0}} \Delta P_B \\ &= (k_{xB})x + (k_{pB})p_B \end{aligned} \quad (3)$$

The coefficients k_{xA} , k_{xB} , k_{pA} and k_{pB} represent the *derivatives* of the function $g_A(X, P_A)$ and $g_B(X, P_B)$ with respect to X and P , respectively. The minus sign in the second of Eqs. (2), because the flow rate *decreases* as the pressure in the piston chamber *increases*.

Assuming the fluid is *incompressible*, the piston velocity can be related to the volumetric flow rates as follows.

$$\boxed{Q_A = A_c \dot{Y}} \quad \text{and} \quad \boxed{Q_B = A_r \dot{Y}} \quad (4)$$

Defining variations from the nominal conditions for each port ($Q = Q_0 + q$, $\dot{Y} = \dot{Y}_0 + \dot{y}$ and $Q_0 = A\dot{Y}_0$), then variations in the piston velocity can be related to changes in the volumetric flow rates as follows.

$$\boxed{q_A = A_c \dot{y}} \quad \text{and} \quad \boxed{q_B = A_r \dot{y}} \quad (5)$$

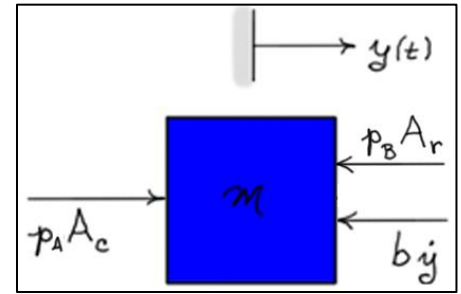
Combining Eqs. (2), (3) and (5) gives the following equations for the pressure changes at each port.

$$\boxed{p_A = (k_{xA}x - A_c \dot{y}) / k_{pA}} \quad \text{and} \quad \boxed{p_B = (-k_{xB}x + A_r \dot{y}) / k_{pB}} \quad (6)$$

Model for Piston Movement

Assuming $X > 0$ (flow is entering port A and leaving port B), **Newton's second law** can be used to write the equation of motion of the load.

$$\boxed{\sum F = p_A A_c - p_B A_r - b \dot{y} = m \ddot{y}} \quad (7)$$



Rearranging the Eq. (7) and **substituting** for the **pressure changes** from Eqs. (6) gives

$$\boxed{m \ddot{y} + \left(b + \frac{A_c^2}{k_{pA}} + \frac{A_r^2}{k_{pB}} \right) \dot{y} = \left(\frac{A_c k_{xA}}{k_{pA}} + \frac{A_r k_{xB}}{k_{pB}} \right) x} \quad (X > 0) \quad (8)$$

If $X < 0$, then $p_A = (k_{xA}x - A_c \dot{y})/k_{pA}$, $p_B = (A_r \dot{y} - k_{xB}x)/k_{pB}$. Substituting these new pressure equations into Newton's law yields the **same form of model** equation as shown in Eq. (8). Note, however, **the coefficients** k_{xA} , k_{xB} , k_{pA} and k_{pB} **will be different** for the two cases, because the nominal pressures (about which the linearization is done) will be different.

Notes

- If we have a **double rod cylinder**, then $A_c = A_r$, so the same model equation holds for motion in both directions. All coefficients will be the same.
- The motion described by Eq. (8) is **second-order, over-damped motion**.
- If the mass of the load is small ($m \approx 0$), then the motion is **first-order**.

Orifice Flow

The volumetric flow rate through a **sharp-edged orifice** can be **approximated** using the equation

$$\boxed{Q = C_d A \sqrt{2 \Delta P / \rho}} \quad (9)$$

Here, A is the orifice area, ρ is the fluid mass density, ΔP is the pressure drop across the orifice, and C_d is a dimensionless discharge coefficient that depends on Reynolds number and the area reduction. Discharge coefficients in the range $0.6 \leq C_d \leq 0.8$ are reasonable for a large range of Reynolds numbers and area reductions.

Using Eq. (9) to *model* the *flow into* and *out of* the cylinder through the control valve, and assuming the *area* of the *orifice* is *proportional* to the *valve spool displacement*, X , Eq. (9) can be written as

$$\boxed{Q_{\text{in}} = C_d \ell X \sqrt{2(P_s - P) / \rho}}$$

$$\boxed{Q_{\text{out}} = C_d \ell X \sqrt{2(P - P_T) / \rho} \approx C_d \ell X \sqrt{2P / \rho}}$$

where ℓ is a characteristic length such that $A = \ell X$. Using these two equations, the coefficients in Eqs. (2) and (3) can be estimated as shown below. *For flow into one of the piston chambers*, the coefficients can be estimated to be

$$k_x = \left(\frac{\partial Q_{\text{in}}}{\partial X} \right)_{X_0, P_0} = \sqrt{\frac{2}{\rho}} C_d \ell \sqrt{P_s - P_0}$$

$$k_p = \left(\frac{\partial Q_{\text{in}}}{\partial P} \right)_{X_0, P_0} = \frac{1}{2} C_d \ell X_0 \sqrt{\frac{2}{\rho}} (P_s - P_0)^{-\frac{1}{2}} = \frac{C_d \ell X_0}{\sqrt{2\rho(P_s - P_0)}}$$

For flow out of one of the chambers, the coefficients can be estimated to be

$$k_x = \left(\frac{\partial Q_{\text{out}}}{\partial X} \right)_{X_0, P_0} = \sqrt{\frac{2}{\rho}} C_d \ell \sqrt{P_0}$$

$$k_p = \left(\frac{\partial Q_{\text{out}}}{\partial P} \right)_{X_0, P_0} = \frac{1}{2} C_d \ell X_0 \sqrt{\frac{2}{\rho}} (P_0)^{-\frac{1}{2}} = \frac{C_d \ell X_0}{\sqrt{2\rho P_0}}$$