

ME 4710 Motion and Control

Compensator Design for ITAE Optimal Response with Pre-filters

(Reference: Dorf & Bishop, Modern Control Systems, Prentice-Hall, 2001)

The forms of *ITAE optimal* transfer functions for *step* and *ramp* inputs are given in Tables 5.6 and 5.7 of Dorf & Bishop's Modern Control Systems. To guarantee ITAE optimal behavior, a compensator can be designed to force the closed-loop transfer function to be of *ITAE optimal form*. The use of *pre-filters* may be required to obtain optimal form.

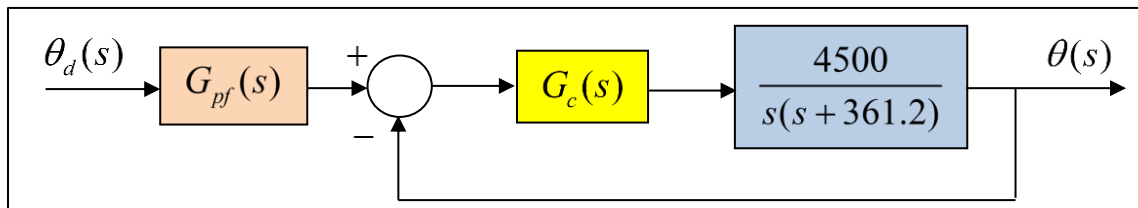
Design Strategy

1. **Step Input:** Assume $\zeta = 0.7$; **Ramp Input:** Assume $\zeta = 1.6$
2. Then *select* ω_n to satisfy the *settling time* specification.
3. Find the compensator parameters to give a *characteristic equation* of the form given in Table 5.6 (step input) or Table 5.7 (ramp input).
4. Design a *pre-filter* to “cancel” the unwanted terms in the numerator of the closed-loop transfer function.

Example

Design a PID compensator that will produce *ITAE optimal, closed-loop step response* with a settling time $T_s = 0.005$ (sec) for the aircraft attitude control system shown in the block diagram.

Use a pre-filter as necessary.



Solution

1. Find ω_n by setting $T_s = 4/\zeta\omega_n = 0.005$ (sec). Assuming $\zeta = 0.7$ (for a step input), $\omega_n = 1143$ (rad/sec).
2. Assuming a PID controller of the form $G_c(s) = \frac{K(s^2 + as + b)}{s}$, the closed loop transfer function (without the pre-filter) is found to be

$$T(s) = \frac{4500K(s^2 + as + b)}{s^2(s + 361.2) + 4500K(s^2 + as + b)}$$

With **PID** control and **no pre-filter**, this system is **type-2**, so it can be **optimized** for **ramp input**. However, in this example, the **optimization** is for **step response**.

3. Equate the **characteristic equation** with that provided by Table 5.6 with $\omega_n = 1143$.

$$\boxed{s^3 + (361.2 + 4500K)s^2 + (4500aK)s + (4500bK) = s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3}$$

Equating the polynomial coefficients and solving for K , a , and b gives

$$K = 0.36423 \quad a = 1713.73 \quad b = 9.110673 \times 10^5$$

or

$$K_p = 624.2 \quad K_I = 331,838 \quad K_D = 0.36423$$

4. **Pre-filter**: To provide ITAE optimal response, choose the pre-filter

$$\boxed{G_{pf}(s) = \frac{(1143)^3}{1639(s^2 + as + b)}}$$

So, the final transfer function has **ITAE optimal form**

$$\frac{\theta_d(s)}{\theta(s)} = \left(\frac{(1143)^3}{1639(s^2 + as + b)} \right) \left(\frac{1639(s^2 + as + b)}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + (1143)^3} \right)$$

$$\Rightarrow \boxed{\frac{\theta_d(s)}{\theta(s)} = \frac{(1143)^3}{s^3 + 2000s^2 + (2.8089 \times 10^6)s + (1143)^3}}$$

5. Closed-loop step response **with** and **without** pre-filter: From the plots below, it is clear the closed loop system **without a pre-filter** has **large overshoot** due to the zeros in the PID compensator. This problem is removed by including a pre-filter. Both systems have settling times slightly larger than 0.005 (sec). Further increases on the choice of ω_n will resolve this issue as well.

6. Error analysis **with a pre-filter**: The system error can be calculated as follows

$$E(s) = \theta_d(s) - \theta(s) = \theta_d(s) - (G_{pf}(s)T(s))\theta_d(s) = (1 - G_{pf}(s)T(s))\theta_d(s)$$

$$= \left(1 - \frac{(1143)^3}{s^3 + 2000s^2 + (2.8089E6)s + (1143)^3} \right) \theta_d(s)$$

So, the error transfer function is

$$\frac{E(s)}{\theta_d(s)} = \frac{s(s^2 + 2000s + 2.8089E6)}{s^3 + 2000s^2 + (2.8089E6)s + (1143)^3}$$

Note the system *with the pre-filter* has a *zero* steady-state error to a *step input* and a *finite* steady-state error to a *ramp input* (typical of a *type-1* system).

7. Error analysis *with no pre-filter*: The system with no pre-filter is a *type-2* system, so it has a *zero* steady-state error to both *step* and *ramp* inputs. Clearly, *some error control is lost when using the pre-filter*.

