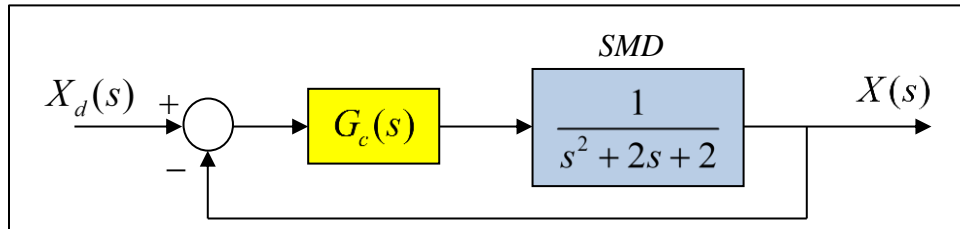


ME 4710 Motion and Control

Frequency Response Design of a Phase-Lag Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the *frequency response design* of a *phase-lag compensator*, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the *desired* and *actual* positions of the mass.



Using proportional control ($G_c(s) = K$), *large* gains are required to control steady-state error to a step input. Unfortunately, large gains produce *undesirable, oscillatory* closed-loop response. Below, a phase-lag compensator is designed to lower the steady-state error without introducing highly oscillatory behavior.

Problem: Design a phase-lag compensator so the closed-loop system has a *steady-state position error* $e_{ss} = 1 - x_{ss} < 0.1$ to a unit step input and a *phase margin* $PM = 45$ (deg). Plot the step response of the resulting closed-loop system.

Frequency Response Design

Step 1: Determine the *compensator gain* required to satisfy the *error specification*.

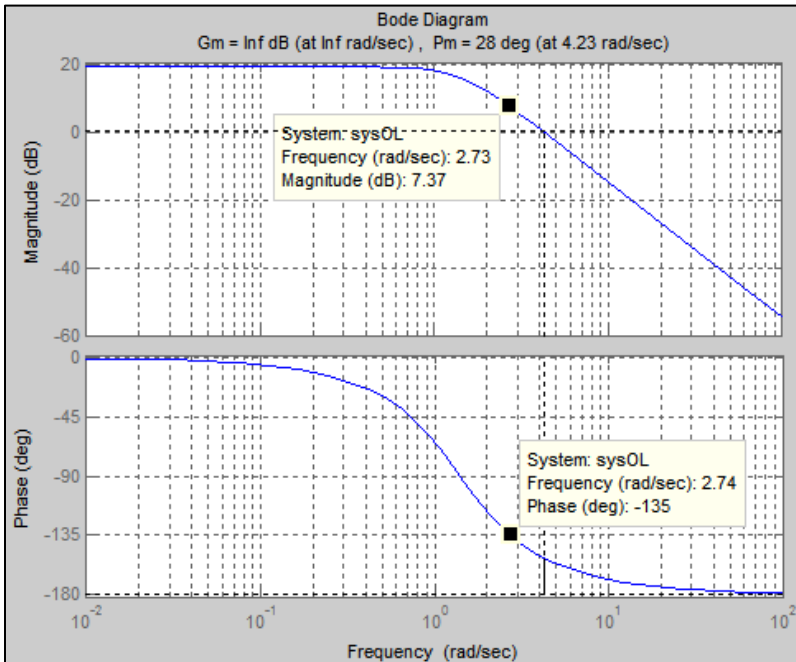
The steady state error can be defined in terms of the loop transfer function as

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{1}{1 + GH(s)} \right] = \frac{1}{1 + K/2} = \frac{1}{1 + K_p} < 0.1$$

To meet the error specification, set $K_p > 9$ and $K > 18$.

Step 2: Evaluate the *phase margin* of the *uncompensated system* with $K = 18$.

Using MATLAB, the phase margin of this (the uncompensated system) is $PM \cong 28$ (deg). See plot below. This is well below the desired phase margin.



Phase Margin of Uncompensated System is 28 (deg) at 4.2 (rad/s)

Step 3: Locate the zero of the compensator

By examination of the Bode diagram, the phase margin requirement would be satisfied at 2.74 (rad/sec). This statement assumes the magnitude plot crosses over the **zero-dB** line at this point. So, **locate the zero** of the compensator **at least one decade below this frequency**, for example, at 0.25 (rad/sec).

Step 4: Locate the pole of the compensator

To make 2.74 (rad/s) the zero-dB crossover point, require approximately **8 dB of attenuation from the compensator**. (See Bode plot of uncompensated system.) Now calculate α by setting $-8 = 20\log(\alpha)$ to find $\alpha = 0.3981$. The first iteration yields the compensator

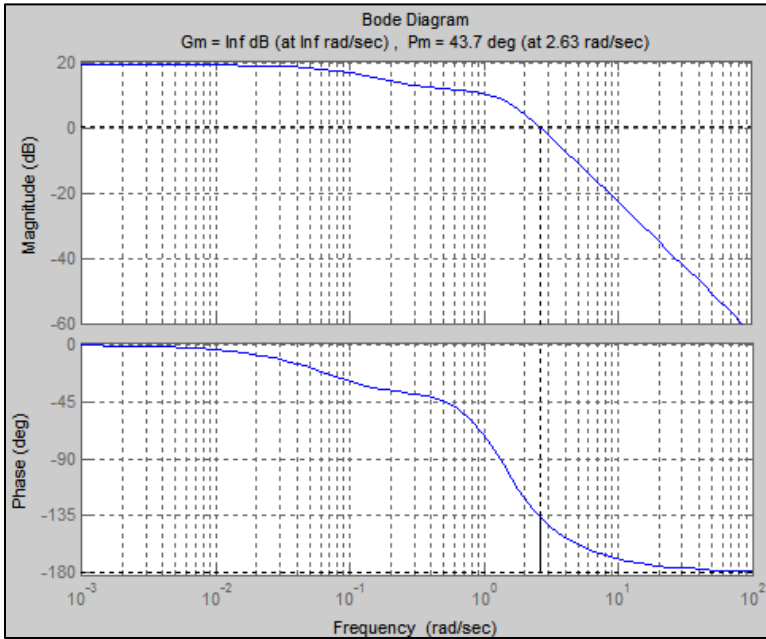
$$G_c(s) = 0.3981 \left[\frac{s + 0.25}{s + 0.0995} \right] \quad (\text{Phase-lag compensator})$$

Step 5: Check the phase margin of the new compensated system.

The Bode diagram of the loop transfer function of the compensated system

$$GH(s) = 0.3981 \left[\frac{s + 0.25}{s + 0.0995} \right] \left[\frac{18}{s^2 + 2s + 2} \right]$$

shows that the **phase margin** is $PM = 43.7$ (deg), satisfying the original requirement. Note the original gain of $K = 18$ is included in the plant transfer function.



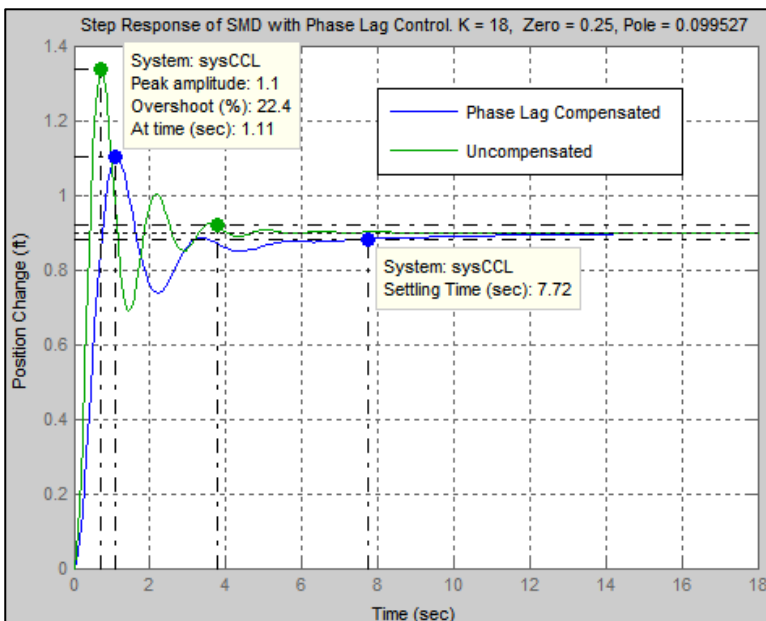
Phase Margin of Compensated System is 43.7 (deg)

Step 6: Repeat steps 3-5 until the *desired phase margin* is obtained.

The new compensated system nearly meets the specification. This result is used in the step response below.

Step 7: Check the step response.

Step response of the uncompensated system shows a large overshoot (about 49%) and low damping with a settling time of 3.78 seconds, while the step response of the phase-lag compensated system shows a smaller overshoot (around 22%) with higher damping and a longer settling time of 7.7 seconds. So, the reduction of overshoot comes at the price of increasing the settling time.



Step Response of Compensated System has *less overshoot* and *more damping* than the Uncompensated System.

Unfortunately, it also has a *longer settling time*. This is typical of integrator-type compensators.

Frequency Response of Compensated Closed Loop System:

The frequency response of the closed loop system is shown below. The ***bandwidth*** of the closed loop system is approximately 4 (rad/s).

