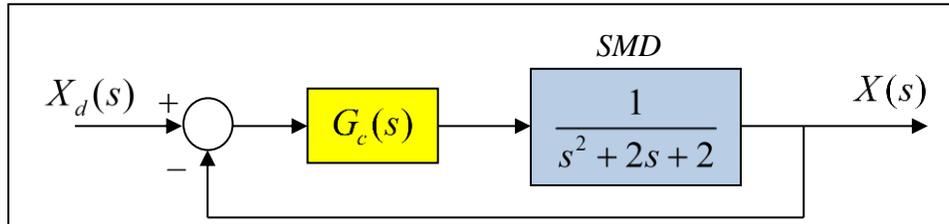


ME 4710 Motion and Control

Root Locus Design of a Phase-Lag Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the *root locus design* of a *phase-lag compensator*, consider the following SMD positioning system controlled by the compensator $G_c(s)$. Here, $X_d(s)$ and $X(s)$ are the *desired* and *actual* positions of the mass.



Using proportional control ($G_c(s) = K$), *large* gains are required to control *steady-state error* for a step input. Unfortunately, *large gains* produce *undesirable, oscillatory* closed-loop response. Below, a phase-lag compensator is designed to lower the steady-state error without introducing highly oscillatory behavior.

Problem: Design a phase-lag compensator so the closed-loop system has a *steady-state step position error* $e_{ss} = 1 - x_{ss} < 0.1$ and *damping factors* for the complex poles of $\zeta \geq 0.4$. Plot the step response of the resulting closed-loop system.

Root Locus Design

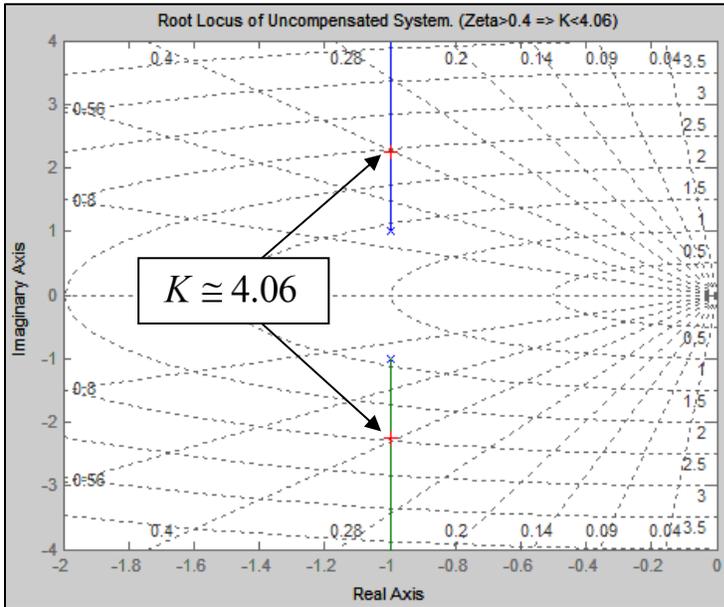
Step 1: Examine *RL diagram* of *uncompensated system*. Find the *gain* required to satisfy the *steady-state error* requirement.

The root locus diagram of the uncompensated system is very simple. The poles of $GH(s)$ are $-1 \pm 1j$. For $K > 0$ the roots move to infinity along the asymptotes at $\sigma_A = -1$. For $K < 0$ the roots move into the break point at -1 and then move along the positive and negative real axis.

For $\zeta \geq 0.4$, we use MATLAB to show that $K \leq 4.06$. See diagram below. Using $K = 4.06$, the steady-state error for the uncompensated system is

$$e_{ss} = \lim_{s \rightarrow 0} \left[\frac{1}{1 + GH(s)} \right] = \frac{1}{1 + (4.06/2)} = 0.33$$

This is clearly higher than the specified value of 0.1. To lower the steady-state error to 0.1, we require $K = 18$.



Root Locus of Uncompensated System. For the damping ratio to be greater than 0.4, the system gain must be less than 4.06.

If $K = 18$ is used, a *phase-lag compensator* can be added to increase the system damping. This will also *increase the settling time* of the system.

Step 2: Evaluate how the *compensator* changes the *RL diagram*.

Here, the loop transfer function $GH(s) = \frac{K \alpha (s + z)}{(s + p)(s^2 + 2s + 2)}$, so the system has asymptotes

at $\phi_A = \pm 90$ (deg) that intersect the real axis at $\sigma_A = \frac{2(-1) - p + z}{2}$. In a phase-lag compensator,

$z > p$, so the asymptotes will be *moved to the right*. However, they will not be moved far, because the *pole* and *zero* are generally located *close to each other*.

Step 3: Try different pole-zero combinations to see effect on RL diagram

Based on the ratio of the uncompensated system gain ($K \approx 4$) and the desired gain ($K \approx 18$), start by picking a pole close to the origin (say $p = 0.1$) and setting

$$\alpha = K_{\text{uncompensated}} / K_{\text{desired}} = 4 / 18 \approx 0.2 \quad \text{and} \quad z = p / \alpha = 0.1 / 0.2 = 0.5$$

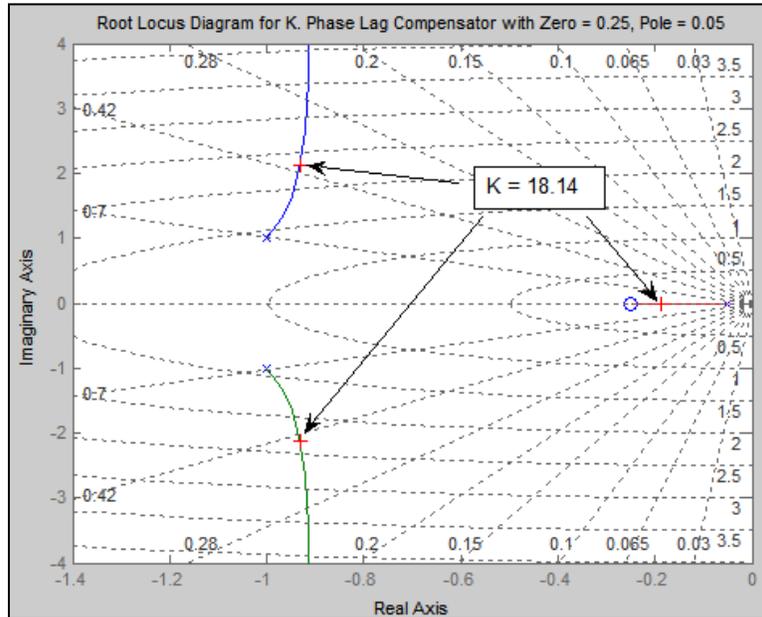
Two questions must now be answered: 1) Can roots be found with $\zeta > 0.4$? 2) Can a *large enough value for K* be chosen so the steady-state error is small? From the root locus diagram, if $\zeta \geq 0.4$, then the maximum K value is approximately 16. *After some iteration* (moving the pole/zero combination closer to the imaginary axis), the following results were found.

For $p=0.05$ and $z=0.25$, the loop transfer function of the compensated system is

$$GH(s) = \frac{0.2K(s+0.25)}{(s+0.05)(s^2+2s+2)},$$

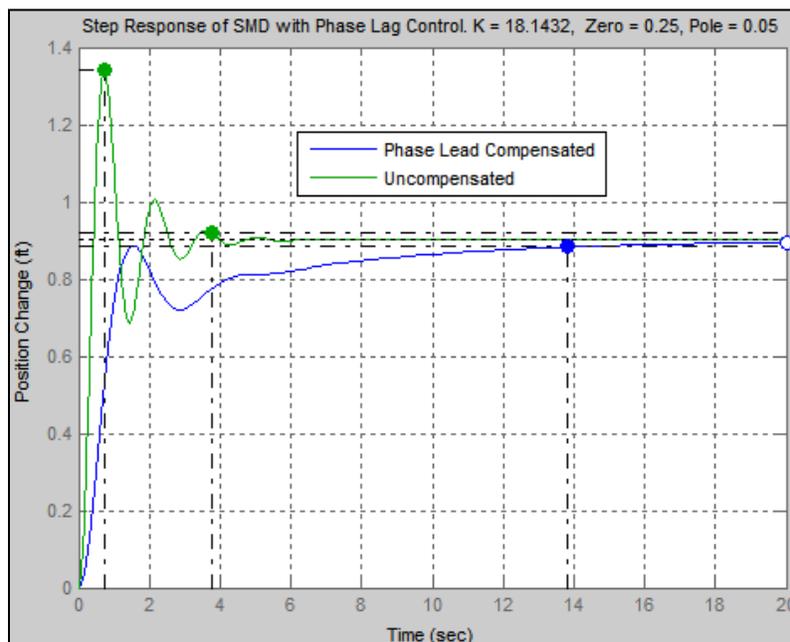
and the root locus diagram yields $K \approx 18$ when the complex

poles have $\zeta \approx 0.4$. See the diagram below.



Step 4: Check the step response.

The step response of the *uncompensated system* with gain $K \approx 18.14$ (same steady-state error as the compensated system) shows a **large overshoot** (49%), low damping, and a **settling time** of approximately 3.8 seconds, while the step response of the compensated system shows no overshoot, higher damping, and a settling time around 13.8 seconds.



If some *overshoot is acceptable* (allowing smaller ζ), the *gain can be increased* in the above design to yield the results shown below for a gain of $K \approx 27.44$. With this gain the system has about 10% overshoot and a settling time of 11 seconds. Note also that with the larger gain, the steady state error is smaller ($e_{ss} \approx 0.07$).

