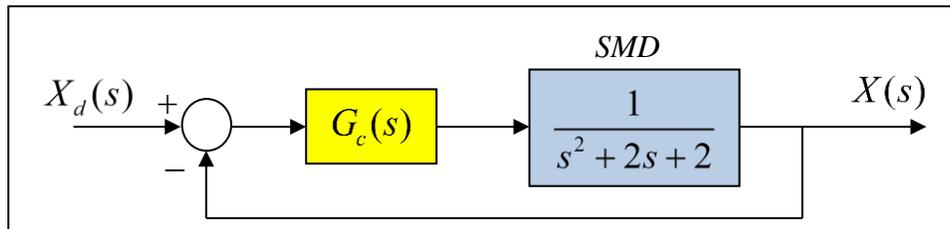


## ME 4710 Motion and Control

### Root Locus Design of a Phase-Lead Compensator for a Spring-Mass-Damper (SMD) Positioning System

To illustrate the *root locus design of a phase-lead compensator*, consider the following SMD positioning system controlled by the compensator  $G_c(s)$ . Here,  $X_d(s)$  and  $X(s)$  are the desired and actual positions of the mass.



Using proportional control ( $G_c(s) = K$ ), *large gains* are required to control *steady-state error* to a step input. Unfortunately, large gains produce *undesirable, oscillatory* closed-loop response. A phase-lead compensator is designed below to *control the steady-state error* and give *desirable transient response*.

**Problem:** Design a phase-lead compensator so the closed-loop system has a settling time  $T_s < 1$  (sec), a damping ratio of the complex roots  $\zeta > 0.5$ , and a small steady-state position error. Plot the step response of the resulting closed-loop system.

#### Root Locus Design:

**Step 1:** Examine *RL diagram* of *uncompensated system*. (same as proportional control)

The root locus of the *uncompensated* system is *very simple*. The poles of  $GH(s)$  are  $-1 \pm 1j$ . For  $K > 0$  the roots move to infinity along the asymptotes at  $\sigma_A = -1$ . For  $K < 0$  the roots move into the break point at  $s = -1$  and then move along the positive and negative real axis.

**Step 2:** *Evaluate* how the compensator *changes the RL diagram*.

The loop transfer function of the compensated system is  $GH(s) = \frac{K(s+z)}{(s+p)(s^2+2s+2)}$ , so the system has asymptotes at  $\phi_A = \pm 90$  (deg) that intersect the real axis at  $\sigma_A = \frac{2(-1) - p + z}{2}$ . **To ensure a settling time of less than 1 second**,  $\sigma_A < -4$ , or a pole-zero separation of  $z - p < -6$ .

As a first design, *assume that*  $z - p = -10$ . Note that this is only a starting point. More separation may be necessary.

**Step 3: Try different pole-zero combinations to see effect on RL diagram**

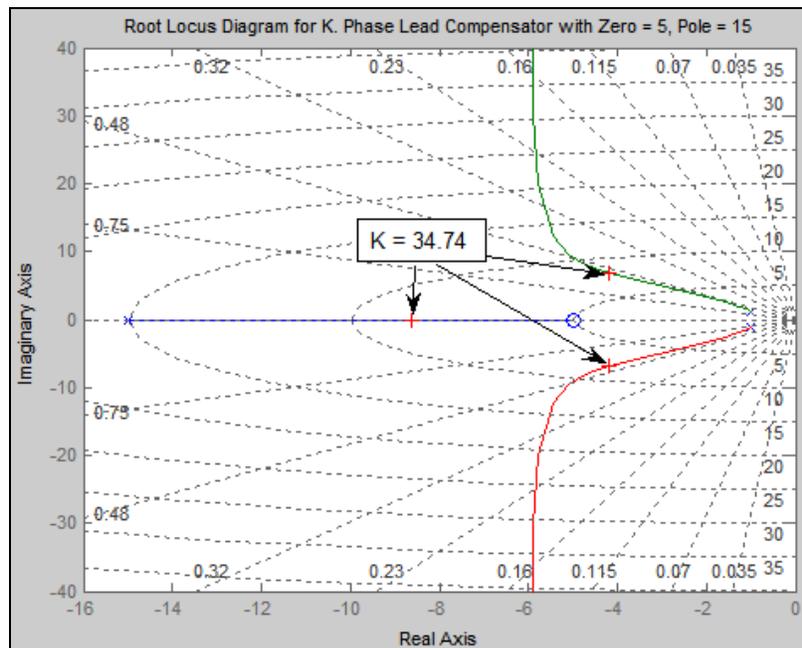
Try first  $z = 5$  and  $p = 15$ , so the loop transfer function of the compensated system is

$$GH(s) = \frac{3K(s + 5)}{(s + 15)(s^2 + 2s + 2)}$$

*Two questions must now be answered.* 1) Can roots be found with  $\zeta > 0.5$ ? 2) Can large enough values for  $K$  be chosen so the steady-state error is small? In this case, the answer to both questions is “yes!” See the results below. If the answer to either question is “no”, then slide the pole-zero combination along the axis to satisfy both requirements. Larger pole-zero separations can also be tried.

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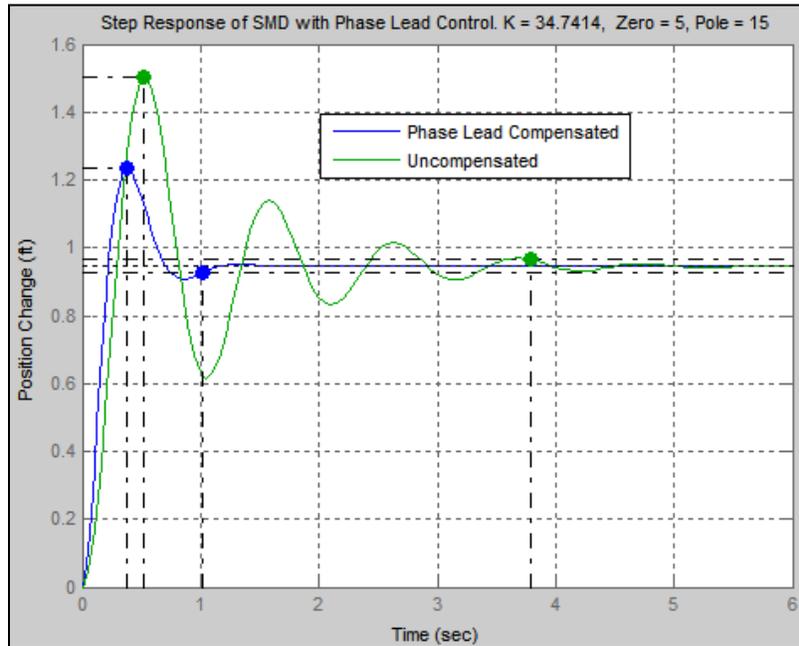
The root locus plot below is for the *compensated system* and the chosen pole locations correspond to  $K \approx 34.74$ . Note the compensator zero (also a zero of the closed loop system) is close to the complex poles, so we expect it to cause larger overshoots than would be expected for  $\zeta = 0.5$ .



**Step 4: Check the step response.**

The step response of the *uncompensated system* with gain  $K \approx 34.74$  (to give the same steady-state error as the compensated system) *shows a large overshoot* (%OS  $\approx 59\%$ ) and *low damping*

with a settling time  $T_s \approx 3.8$  (sec), while the step response of the *compensated system* shows a *smaller overshoot* (%OS  $\approx 30\%$ ) with *higher damping* with a settling time  $T_s \approx 1$  (sec).



As an alternate design, the plots below show the root locus and step response plots for a phase lead compensator with  $z = 3$  and  $p = 25$ . The root locus diagram shows the location of the closed loop poles for  $K \approx 35$ . The step response of the compensated system for  $K \approx 35$  shows a 14% overshoot and a settling time of 0.7 seconds. Note the location of the real pole nearly cancels the compensator's zero hence reducing its effect on the system overshoot.

