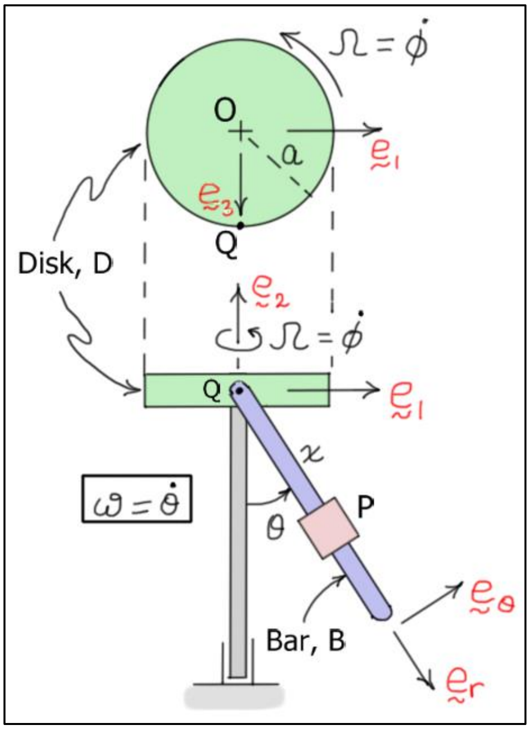
ME 5550 Intermediate Dynamics
Example: Slider on a Rotating Bar



Problem: Given the position coordinates (x, ϕ, θ), find ${}^R\underset{\sim}{v}_P$ the velocity of P and ${}^R\underset{\sim}{a}_P$ the acceleration of P.

Solution:

Reference frames: $\begin{cases} D:(\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3) \\ B:(\underset{\sim}{e}_r, \underset{\sim}{e}_\theta, \underset{\sim}{e}_3) \end{cases}$

Angular velocity of B:

$$\boxed{\begin{aligned} {}^R\underset{\sim}{\omega}_B &= {}^R\underset{\sim}{\omega}_D + {}^D\underset{\sim}{\omega}_B \\ &= \Omega \underset{\sim}{e}_2 + \omega \underset{\sim}{e}_3 \end{aligned}}$$

Angular acceleration of B:

$${}^R\underset{\sim}{\alpha}_B = \frac{{}^R d}{dt}\left({}^R\underset{\sim}{\omega}_B\right) = \dot{\Omega}\underset{\sim}{e}_2 + \dot{\omega}\underset{\sim}{e}_3 + \omega\dot{\underset{\sim}{e}}_3 = \dot{\Omega}\underset{\sim}{e}_2 + \dot{\omega}\underset{\sim}{e}_3 + \omega\left(\Omega\underset{\sim}{e}_2 \times \underset{\sim}{e}_3\right)$$

$$\boxed{{}^R\underset{\sim}{\alpha}_B = \omega\,\Omega\,\underset{\sim}{e}_1 + \dot{\Omega}\,\underset{\sim}{e}_2 + \dot{\omega}\,\underset{\sim}{e}_3}$$

Velocity of P:

$$\boxed{{}^R\underset{\sim}{v}_P = {}^R\underset{\sim}{v}_{\hat{P}} + {}^B\underset{\sim}{v}_P}$$

(P ***moves*** on B and $\hat{P}$ is ***fixed*** on B)

Here,

$${}^R\underset{\sim}{v}_{\hat{P}} = {}^R\underset{\sim}{v}_Q + {}^R\underset{\sim}{v}_{\hat{P}/Q} = a\Omega\underset{\sim}{e}_1 + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{r}_{\hat{P}/Q}\right) = a\Omega\underset{\sim}{e}_1 + \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ xS_\theta & -xC_\theta & 0 \end{vmatrix}$$

$$\Rightarrow \boxed{{}^R\underset{\sim}{v}_{\hat{P}} = (a\Omega + x\omega C_\theta)\underset{\sim}{e}_1 + (x\omega S_\theta)\underset{\sim}{e}_2 - (x\Omega S_\theta)\underset{\sim}{e}_3}$$

$$\boxed{{}^B\underset{\sim}{v}_P = \dot{x}\underset{\sim}{e}_r = \dot{x}(S_\theta\underset{\sim}{e}_1 - C_\theta\underset{\sim}{e}_2)}$$

Combining the above results gives the result for ${}^R\underset{\sim}{v}_P$.

$$\boxed{{}^R\underset{\sim}{v}_P = (a\Omega + x\omega C_\theta + \dot{x}S_\theta)\underset{\sim}{e}_1 + (x\omega S_\theta - \dot{x}C_\theta)\underset{\sim}{e}_2 - (x\Omega S_\theta)\underset{\sim}{e}_3}$$

Acceleration of P:

$$\boxed{{}^R\underset{\sim}{a}_P = {}^R\underset{\sim}{a}_{\hat{P}} + {}^B\underset{\sim}{a}_P + 2\left({}^R\underset{\sim}{\omega}_B \times {}^B\underset{\sim}{v}_P\right)}$$

Here,

$${}^R\underset{\sim}{a}_{\hat{P}} = {}^R\underset{\sim}{a}_Q + {}^R\underset{\sim}{a}_{\hat{P}/Q}$$

$$\boxed{{}^R\underset{\sim}{a}_Q = a\dot{\Omega}\,\underset{\sim}{e}_1 - a\Omega^2\,\underset{\sim}{e}_3}$$

$${}^R\underset{\sim}{a}_{\hat{P}/Q} = ({}^R\underset{\sim}{\alpha}_B \times \underset{\sim}{r}_{\hat{P}/Q}) + ({}^R\underset{\sim}{\omega}_B \times {}^R\underset{\sim}{v}_{\hat{P}/Q}) = \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ \omega\Omega & \dot{\Omega} & \dot{\omega} \\ xS_\theta & -xC_\theta & 0 \end{vmatrix} + \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ x\omega C_\theta & x\omega S_\theta & -x\Omega S_\theta \end{vmatrix}$$

$$\boxed{\begin{aligned} {}^R\underset{\sim}{a}_{\hat{P}/Q} = &\left(x\dot{\omega}C_\theta - x\Omega^2 S_\theta - x\omega^2 S_\theta\right)\underset{\sim}{e}_1 + \left(x\dot{\omega}S_\theta + x\omega^2 C_\theta\right)\underset{\sim}{e}_2 - \\ &\left(x\omega\Omega C_\theta + x\dot{\Omega}S_\theta + x\omega\Omega C_\theta\right)\underset{\sim}{e}_3 \end{aligned}}$$

$$\boxed{{}^B\underset{\sim}{a}_P = \ddot{x}\,\underset{\sim}{e}_r = \ddot{x}\left(S_\theta\,\underset{\sim}{e}_1 - C_\theta\,\underset{\sim}{e}_2\right)}$$

$$2\,{}^R\underset{\sim}{\omega}_B \times {}^B\underset{\sim}{v}_P = 2\begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ \dot{x}S_\theta & -\dot{x}C_\theta & 0 \end{vmatrix}$$

$$\boxed{2\,{}^R\underset{\sim}{\omega}_B \times {}^B\underset{\sim}{v}_P = 2\left[(\dot{x}\omega C_\theta)\underset{\sim}{e}_1 + (\dot{x}\omega S_\theta)\underset{\sim}{e}_2 - (\dot{x}\Omega S_\theta)\underset{\sim}{e}_3\right]}$$

Combining all the above terms gives the result for ${}^R\underset{\sim}{a}_P$.

$$\boxed{\begin{aligned} {}^R\underset{\sim}{a}_P = &\left(a\dot{\Omega} + x\dot{\omega}C_\theta - xS_\theta(\omega^2 + \Omega^2) + \ddot{x}S_\theta + 2\dot{x}\omega C_\theta\right)\underset{\sim}{e}_1 + \\ &\left(x\dot{\omega}S_\theta + x\omega^2 C_\theta - \ddot{x}C_\theta + 2\dot{x}\omega S_\theta\right)\underset{\sim}{e}_2 - \\ &\left(a\Omega^2 + x\dot{\Omega}S_\theta + 2x\omega\Omega C_\theta + 2\dot{x}\Omega S_\theta\right)\underset{\sim}{e}_3 \end{aligned}}$$