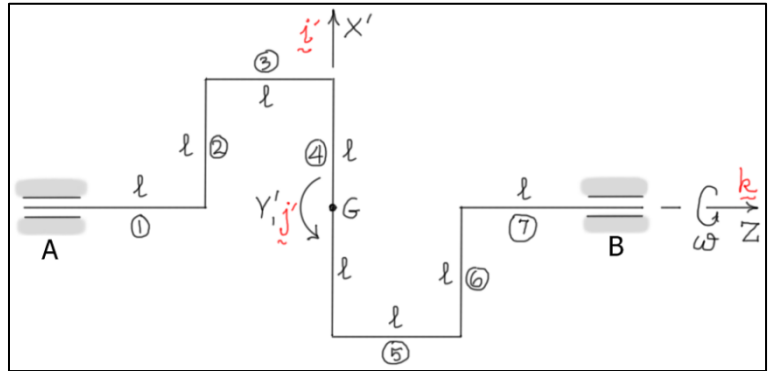


ME 5550 Intermediate Dynamics

Bearing Loads on a Simple Crank Shaft

The figure shows a *simple crank shaft* consisting of *seven* segments, each considered to be a *slender bar*. Each segment of length ℓ has mass m . There are six segments of length ℓ and one segment of length 2ℓ (segment 4).



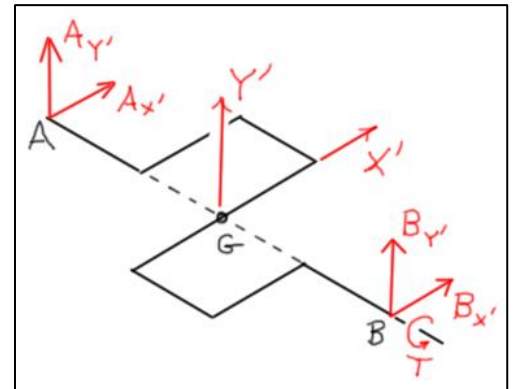
The *mass center* of the system is G and is located on the axis of rotation. In previous notes, \underline{H}_G the *angular momentum* of the system was found to be

$$\underline{H}_G = 2m\ell^2\omega\tilde{i}' + \left(\frac{10}{3}\right)m\ell^2\omega\tilde{k}$$

The *bearing loads* at A and B can be found by applying the *Newton/Euler equations of motion* to the *free-body* diagram shown at the right.

$$\begin{aligned} \sum \underline{F} &= m^R \underline{a}_G = \underline{0} \\ \sum \underline{M}_A &= (\underline{I}_G \cdot {}^R \underline{\alpha}_B) + ({}^R \underline{\omega}_B \times \underline{H}_G) + (\underline{r}_{G/A} \times m^R \underline{a}_G) \\ &= (\underline{I}_G \cdot {}^R \underline{\alpha}_B) + ({}^R \underline{\omega}_B \times \underline{H}_G) \end{aligned}$$

The terms on the left and right sides of the moment equation can be calculated as follows.



$$\sum \underline{M}_A = (\underline{r}_{B/A} \times \underline{F}_B) + T\tilde{k} = -(4\ell B_{Y'})\tilde{i}' + (4\ell B_{X'})\tilde{j}' + T\tilde{k}$$

$$\underline{I}_G \cdot {}^R \underline{\alpha}_B = 2m\ell^2\dot{\omega}\tilde{i}' + \left(\frac{10}{3}\right)m\ell^2\dot{\omega}\tilde{k}$$

$${}^R \underline{\omega}_B \times \underline{H}_G = \omega\tilde{k} \times \underline{H}_G = 2m\ell^2\omega^2\tilde{j}'$$

Substituting these results into the equations above gives the following equations of motion.

Force Equations:

$$\begin{aligned} A_{X'} + B_{X'} &= 0 \\ A_{Y'} + B_{Y'} &= 0 \end{aligned}$$

Moment Equations:

$$\begin{aligned} -4\ell B_{Y'} &= 2m\ell^2 \dot{\omega} \\ 4\ell B_{X'} &= 2m\ell^2 \omega^2 \\ T &= \frac{10}{3} m\ell^2 \dot{\omega} \end{aligned}$$

Solving these equations gives the following results.

$$\begin{aligned} B_{X'} &= \frac{1}{2} m\ell \omega^2 = -A_{X'} \\ B_{Y'} &= -\frac{1}{2} m\ell \dot{\omega} = -A_{Y'} \\ T &= \frac{10}{3} m\ell^2 \dot{\omega} \end{aligned}$$

Notes:

1. Even if the system is rotating at a **constant rate** (so that $\dot{\omega} = 0$), the ***in-plane bearing loads*** ($A_{X'}$ and $B_{X'}$) are still ***non-zero*** due to the ***in-plane asymmetry*** of the crank shaft.
2. The magnitudes of these forces ***increase*** with the ***square*** of the ***rotational speed***.