ME 5550 Intermediate Dynamics
Degrees of Freedom of Mechanical Systems

System Configuration and Generalized Coordinates

The ***configuration*** of a mechanical system is defined as the ***position*** of each of the bodies within the system at any time. In general, both coordinates of ***translation*** and ***rotation*** are needed to describe the position of a rigid body. Taken all together, the coordinates are called ***generalized coordinates***. For the ***simple pendulum*** shown, x_G and y_G are ***coordinates*** of ***translation*** that describe the ***location*** of the mass center of the bar, and θ is a ***coordinate*** of ***rotation*** that describes the ***orientation*** of the bar.

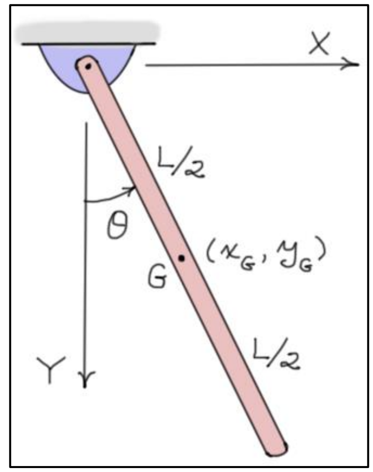

Fig. 1 Simple Pendulum

Typically, the ***generalized coordinates*** used to define the ***configuration*** of the mechanical system form a ***dependent*** set. That is, the coordinates ***cannot all be chosen independently*** of the others. For example, for the simple pendulum of Fig.1, x_G, y_G, and θ form a ***dependent set*** of coordinates. The following equations can be used to relate the three.

$$\boxed{x_G = \tfrac{L}{2}\sin(\theta)} \qquad \boxed{y_G = \tfrac{L}{2}\cos(\theta)}$$

Given the ***value*** of ***one*** of the ***coordinates***, these equations can be used to compute the ***values*** of the ***other two coordinates***. So, only ***one*** of the coordinates is needed to describe the position of the system. ***Any pair*** of these coordinates forms a ***dependent set***.

Generalized Coordinates and Degrees of Freedom

The number of ***degrees of freedom*** (DOF) of a mechanical system is defined as the ***minimum number*** of ***generalized coordinates*** necessary to define the configuration of the system. For a set of generalized coordinates to be ***minimum*** in ***number***, the coordinates must form an ***independent set***. Fig. 2 below shows ***examples*** of one, two, and three degree-of-freedom ***planar systems***.

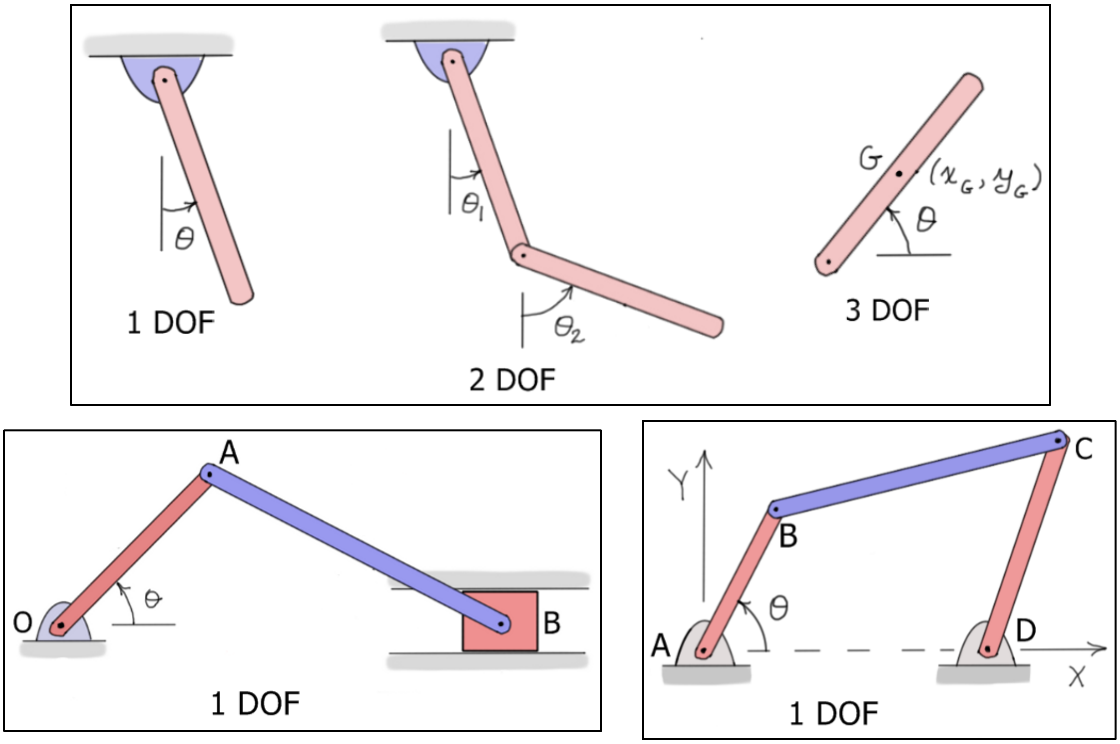

Fig. 2 Examples of One, Two, and Three Degree of Freedom Planar Systems

For ***mechanical systems*** that consist of a series of ***interconnected bodies***, it may not be obvious ***how many degrees of freedom*** the system possesses. For these systems, the number of degrees of freedom can be found by first ***calculating*** the number of degrees of freedom the system would possess if the motions of all the bodies were ***unrestricted*** and then ***subtracting*** degrees of freedom ***removed*** by the ***constraints*** on the motion. For example, for the slider crank mechanism shown in Fig. 2, the number of degrees of freedom can be calculated in the following ways:

a) Counting 3 Bodies: (crank, slider, and connecting bar)

3 Bodies @ 3 DOF	$= 3\times 3$	= 9 DOF (possible)
3 Pin Joints	$= 3\times 2$	= 6 constraints
1 Slider Joint	$= 1\times 2$	= 2 constraints
Total DOF	$= 9-6-2$	= 1 DOF

b) Counting 2 Bodies: (crank and connecting bar)

2 Bodies @ 3 DOF	$= 2\times 3$	= 6 DOF (possible)
2 Pin Joints	$= 2\times 2$	= 4 constraints
1 Pin/Slider Joint	$= 1\times 1$	= 1 constraint
Total DOF	$= 6-4-1$	= 1 DOF