

ME 5550 Intermediate Dynamics

Partial Velocities and Partial Angular Velocities

(Reference: T. R. Kane and D. A. Levinson, *Dynamics, Theory and Applications*, McGraw-Hill, 1985.)

Partial Velocities

If ${}^R \underline{v}_P$ the **velocity** of some point P within a mechanical system can be written in terms of a set of **generalized coordinates** q_i ($i=1, \dots, n$) and their **time derivatives** \dot{q}_i ($i=1, \dots, n$), then the **partial velocities** of P are defined to be the **partial derivatives** of ${}^R \underline{v}_P$ with respect to the \dot{q}_i ($i=1, \dots, n$). That is,

$$\boxed{\frac{\partial {}^R \underline{v}_P}{\partial \dot{q}_i} \quad (i=1, \dots, n)} \quad (1)$$

These vectors represent the **changes** in the **velocity** of P resulting from **changes** in each of \dot{q}_i ($i=1, \dots, n$) the time derivatives of the coordinates. It can be shown that they also represent the **changes** in \underline{r}_P the **position** of P resulting from **changes** each of q_i ($i=1, \dots, n$) the generalized coordinates. Note that, in general, ${}^R \underline{v}_P$ can be written in terms of the partial velocities as follows.

$$\boxed{{}^R \underline{v}_P = \sum_{i=1}^n \left(\frac{\partial \underline{r}_P}{\partial q_i} \right) \dot{q}_i + \frac{\partial \underline{r}_P}{\partial t} = \sum_{i=1}^n \left(\frac{\partial {}^R \underline{v}_P}{\partial \dot{q}_i} \right) \dot{q}_i + \frac{\partial \underline{r}_P}{\partial t}} \quad (2)$$

Here, the term $\frac{\partial \underline{r}_P}{\partial t}$ accounts for position vectors that depend **explicitly on time** (e.g. some specified motion within the system).

Partial Angular Velocities

Similarly, if the **angular velocity** of a body B within a mechanical system can be written in terms of a set of **generalized coordinates** q_i ($i=1, \dots, n$) and their **time derivatives** \dot{q}_i ($i=1, \dots, n$), then the **partial angular velocities** of B are defined to be the **partial derivatives** of ${}^R \underline{\omega}_B$ with respect to the \dot{q}_i ($i=1, \dots, n$).

$$\boxed{\frac{\partial {}^R \underline{\omega}_B}{\partial \dot{q}_i} \quad (i=1, \dots, n)} \quad (3)$$

These vectors represent the *changes* in the **angular velocity** of a body resulting from *changes* in the \dot{q}_i ($i=1, \dots, n$). In general, ${}^R\omega_B$ can be written in terms of the partial velocities as follows.

$$\boxed{{}^R\omega_B = \sum_{i=1}^n \left(\frac{\partial {}^R\omega_B}{\partial \dot{q}_i} \right) \dot{q}_i + ({}^R\omega_B)_t} \quad (4)$$

Here, $({}^R\omega_B)_t$ is that part of the angular velocity vector that *depends explicitly on time*.

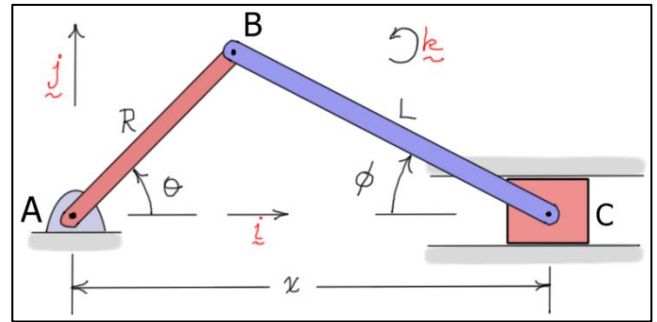
Example

Consider the *slider-crank mechanism* shown in the diagram. It can be shown that the **velocity** of piston C and the **angular velocity** of connecting rod BC can be written as

$$\boxed{{}^R v_C = -R \left[\sin(\theta) + \left(\frac{\cos(\theta) \sin(\phi)}{\cos(\phi)} \right) \right] \dot{\theta} \tilde{i}}$$

and

$$\boxed{{}^R \omega_{BC} = - \left(\frac{R \cos(\theta)}{L \cos(\phi)} \right) \dot{\theta} \tilde{k}}$$



Using these expressions, the *partial velocity* of C and *partial angular velocity* of connecting rod BC with respect to $\dot{\theta}$ are easily identified to be as follows.

$$\boxed{\frac{\partial {}^R v_C}{\partial \dot{\theta}} = -R \left[\sin(\theta) + \left(\frac{\cos(\theta) \sin(\phi)}{\cos(\phi)} \right) \right] \tilde{i}}$$

$$\boxed{\frac{\partial {}^R \omega_{BC}}{\partial \dot{\theta}} = - \left(\frac{R \cos(\theta)}{L \cos(\phi)} \right) \tilde{k}}$$