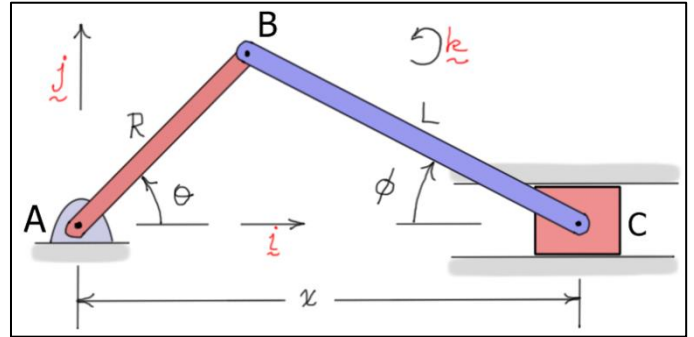


ME 5550 Intermediate Dynamics

Partial Velocities and the Slider Crank Mechanism

System Configuration

The figure shows a simple *slider crank mechanism* with no offset. Given the lengths of the links (R, L), the *configuration* of the system at any instant of time can be given by one or all of the *generalized coordinates* $(q_i) = (\theta, \phi, x)$.



These coordinates are *not independent*.

The following set of *constraint equations* can be used to relate the three coordinates.

$$\begin{cases} RS_{\theta} - LS_{\phi} = 0 \\ RC_{\theta} + LC_{\phi} - x = 0 \end{cases} \quad (1)$$

Here, S_{θ} and S_{ϕ} represent the sines of the angles, and C_{θ} and C_{ϕ} represent their cosines.

Partial Angular Velocities of the Links

Using the angles shown in the diagram, the *angular velocities* of the crank and connecting rod can be written as $\omega_{AB} = \dot{\theta} \tilde{k}$ and $\omega_{BC} = -\dot{\phi} \tilde{k}$. Using these expressions, the *partial angular velocities* of the links can be written as follows.

$$\begin{cases} \frac{\partial \omega_{AB}}{\partial \dot{\theta}} = \tilde{k} \\ \frac{\partial \omega_{BC}}{\partial \dot{\phi}} = -\tilde{k} \end{cases} \quad (2)$$

If the first of Eqs. (1) is differentiated, the angular rates $\dot{\theta}$ and $\dot{\phi}$ can be related as follows.

$$R\dot{\theta}C_{\theta} = L\dot{\phi}C_{\phi} \quad (3)$$

Using this result, another set of *partial angular velocities* can be defined as follows.

$$\frac{\partial \omega_{AB}}{\partial \dot{\phi}} = \frac{\partial}{\partial \dot{\phi}} (\dot{\theta} \tilde{k}) = \frac{\partial}{\partial \dot{\phi}} \left[\left(\frac{LC_{\phi}}{RC_{\theta}} \right) \dot{\phi} \tilde{k} \right] = \left(\frac{LC_{\phi}}{RC_{\theta}} \right) \tilde{k} \quad (4)$$

$$\frac{\partial \omega_{BC}}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}} (-\dot{\phi} \tilde{k}) = \frac{\partial}{\partial \dot{\theta}} \left[- \left(\frac{RC_{\theta}}{LC_{\phi}} \right) \dot{\theta} \tilde{k} \right] = - \left(\frac{RC_{\theta}}{LC_{\phi}} \right) \tilde{k} \quad (5)$$

Eq. (4) indicates how *changes* in the **angular velocity** of the **connecting rod BC** affect the **angular velocity** of the **crank AB**, and Eq. (5) indicates how *changes* in the **angular velocity** of the **crank AB** affect the **angular velocity** of the **connecting rod BC**.

Partial Velocities of the Slider

The **velocity** of the slider can be written most simply as $\boxed{v_C = \dot{x} \hat{i}}$. From this result, the following **partial velocity** can be defined.

$$\boxed{\frac{\partial v_C}{\partial \dot{x}} = \hat{i}} \quad (6)$$

However, the **velocity** of the **slider** can also be written in terms of the crank or connecting bar angular rates using the **relative velocity equation** as follows.

$$\boxed{v_C = v_B + v_{C/B}} \quad (7)$$

Here,

$$v_B = v_{B/A} = \omega_{AB} \times r_{B/A} = \dot{\theta} \hat{k} \times R(C_\theta \hat{i} + S_\theta \hat{j}) \Rightarrow \boxed{v_B = R\dot{\theta}(-S_\theta \hat{i} + C_\theta \hat{j})}$$

$$v_{C/B} = \omega_{BC} \times r_{C/B} = -\dot{\phi} \hat{k} \times L(C_\phi \hat{i} - S_\phi \hat{j}) \Rightarrow \boxed{v_{C/B} = -L\dot{\phi}(S_\phi \hat{i} + C_\phi \hat{j})}$$

Substituting these results into Eq. (7) gives the following.

$$\boxed{v_C = (-R\dot{\theta}S_\theta - L\dot{\phi}S_\phi) \hat{i} + (R\dot{\theta}C_\theta - L\dot{\phi}C_\phi) \hat{j} = (-R\dot{\theta}S_\theta - L\dot{\phi}S_\phi) \hat{i}} \quad (8)$$

Note the \hat{j} component is **zero** as a result of Eq. (3).

Using Eqs. (3) and (8), the following **partial velocities** can now be defined.

$$v_C = -\dot{\theta} \left[RS_\theta + LS_\phi \left(\frac{RC_\theta}{LC_\phi} \right) \right] \hat{i} \Rightarrow \boxed{\frac{\partial v_C}{\partial \dot{\theta}} = -R \left[S_\theta + \left(\frac{C_\theta S_\phi}{C_\phi} \right) \right] \hat{i}} \quad (9)$$

$$v_C = -\dot{\phi} \left[RS_\theta \left(\frac{LC_\phi}{RC_\theta} \right) + LS_\phi \right] \hat{i} \Rightarrow \boxed{\frac{\partial v_C}{\partial \dot{\phi}} = -L \left[S_\phi + \left(\frac{C_\phi S_\theta}{C_\theta} \right) \right] \hat{i}} \quad (10)$$