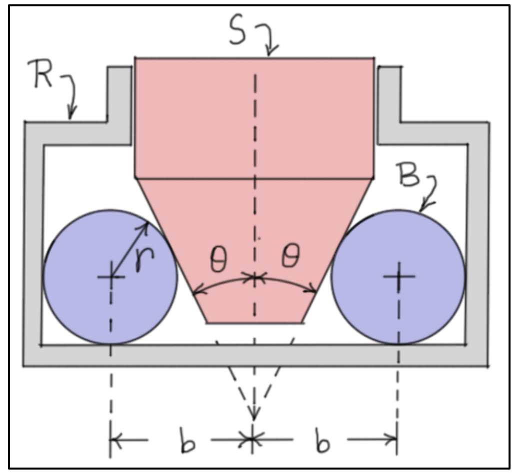
ME 5550 Intermediate Dynamics: Thrust Bearing Example

(Reference: Kane and Levinson, *Dynamics: Theory and Applications*, McGraw-Hill, 1985.

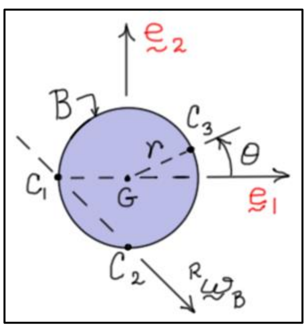
Problem:

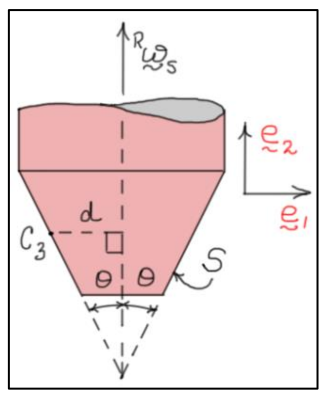
Given the thrust bearing in the diagram, show that for ***pure rolling*** between the shaft S and the bearing B, it is required that $\boxed{b = \dfrac{r(1+S_\theta)}{C_\theta - S_\theta}}$. Note that ***pure rolling*** occurs between S and B if ***no slipping*** occurs and if ${}^B\underset{\sim}{\omega}_S$ the angular velocity of the shaft (S) relative to the bearing (B) is ***parallel*** to the ***common tangent plane*** between S and B. Assume also that ***no slip*** occurs between the bearing and the race (R).



Solution:

1. Consider the separate diagrams below of the shaft and the bearing. Points C_1 and C_2 represent the contact points between the bearing and the race (R), and C_3 represents the contact point between the shaft and the bearing. The unit vectors $\underset{\sim}{e}_1$ and $\underset{\sim}{e}_2$ represent the horizontal and vertical directions in the plane defined by these three points. To complete the unit vector set, a third unit vector is defined as $\underset{\sim}{e}_3 = \underset{\sim}{e}_1 \times \underset{\sim}{e}_2$.





Given this set-up and the ***no-slip conditions*** at points C_1, C_2, and C_3, the angular velocities of the shaft and the bearing may be written as

$$\boxed{{}^R\underset{\sim}{\omega}_S = {}^R\omega_S\,\underset{\sim}{e}_2} \qquad \boxed{{}^R\underset{\sim}{\omega}_B = {}^R\omega_B\left(\tfrac{\sqrt{2}}{2}\underset{\sim}{e}_1 - \tfrac{\sqrt{2}}{2}\underset{\sim}{e}_2\right)} \qquad (1)$$

2. The ***angular velocities*** of the shaft and the bearing in R can be ***related*** by calculating the velocity of the contact point C_3. In this process, advantage is taken of the fact that the velocities of the points C_1 and C_2 relative to R are zero due to the ***no-slip*** condition.

$$ {}^R\underset{\sim}{v}_{C_3} = {}^R\underset{\sim}{v}_{C_1} + {}^R\underset{\sim}{v}_{C_3/C_1} = 0 + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{r}_{C_3/C_1} \right) $$

$$ = {}^R\omega_B \left(\tfrac{\sqrt{2}}{2} \underset{\sim}{e}_1 - \tfrac{\sqrt{2}}{2} \underset{\sim}{e}_2 \right) \times \left(r(1+C_\theta)\underset{\sim}{e}_1 + rS_\theta \underset{\sim}{e}_2 \right) \qquad (C_3 \text{ is a point on the } \textbf{\textit{bearing}}) $$

$$ \Rightarrow \boxed{ {}^R\underset{\sim}{v}_{C_3} = \tfrac{\sqrt{2}}{2} r \left(S_\theta + (1+C_\theta) \right) {}^R\omega_B \, \underset{\sim}{e}_3 } $$

Also,

$$ \boxed{ {}^R\underset{\sim}{v}_{C_3} = d\, {}^R\omega_S \, \underset{\sim}{e}_3 = \left(b - r\, C_\theta \right) {}^R\omega_S \, \underset{\sim}{e}_3 } \qquad (C_3 \text{ is a point on the } \textbf{\textit{shaft}}) $$

Assuming there is ***no-slip*** between the ***bearing*** and the ***shaft***, these two velocities must be equal. Setting the two equal leads to the conclusion that

$$ \boxed{ {}^R\omega_S = \tfrac{\sqrt{2}}{2} \left[\frac{r\left(1 + S_\theta + C_\theta\right)}{\left(b - rC_\theta\right)} \right] {}^R\omega_B } \tag{2} $$

3. The ***angular velocities*** of the shaft and the bearing can also be related using the ***summation rule*** for angular velocities. That is,

$$ \boxed{ {}^R\underset{\sim}{\omega}_S = {}^R\underset{\sim}{\omega}_B + {}^B\underset{\sim}{\omega}_S } \tag{3} $$

Here, ${}^R\underset{\sim}{\omega}_B$ the angular velocity of the bearing is given by Eq. (1), ${}^R\underset{\sim}{\omega}_S$ the angular velocity of the shaft is given by Eqs. (1) and (2), and for ***pure rolling*** between *S* and *B*,

$$ \boxed{ {}^B\underset{\sim}{\omega}_S = {}^B\omega_S \left(-S_\theta \underset{\sim}{e}_1 + C_\theta \underset{\sim}{e}_2 \right) }. \tag{4} $$

Substituting from Eqs. (1), (2), and (4) into Eq. (3) gives the following two scalar equations.

$$ \boxed{ \begin{aligned} &\tfrac{\sqrt{2}}{2}\, {}^R\omega_B - S_\theta \, {}^B\omega_S = 0 \\ -&\tfrac{\sqrt{2}}{2}\, {}^R\omega_B + C_\theta \, {}^B\omega_S = \tfrac{\sqrt{2}}{2} \left[\frac{r\left(1 + S_\theta + C_\theta\right)}{\left(b - rC_\theta\right)} \right] {}^R\omega_B \end{aligned} } \tag{5} $$

Multiplying the first equation by C_θ and the second by S_θ, adding the two equations, and simplifying gives the desired result.

$$ \boxed{ b = \frac{r\left(1 + S_\theta\right)}{C_\theta - S_\theta} } $$