

## ME 5550 Intermediate Dynamics

### Principle of Virtual Work

#### Principle of Virtual Work

If a mechanical system whose *configuration* is defined by a set of *independent generalized coordinates*  $q_k$  ( $k=1,\dots,n$ ) is in *static equilibrium*, then the *generalized force* associated with each of the generalized coordinates  $q_k$  ( $k=1,\dots,n$ ) must be *zero*. That is,

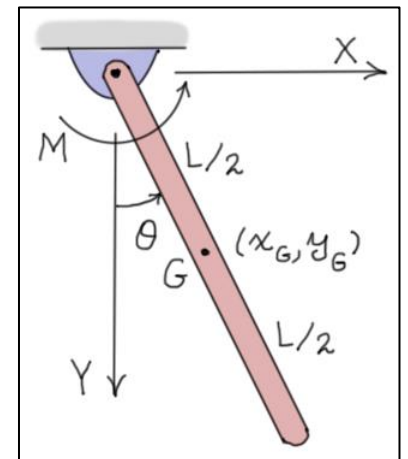
$$\boxed{F_{q_k} = 0} \quad (k = 1, \dots, n)$$

Recall that if a system has “ $n$ ” *degrees of freedom*, then “ $n$ ” independent generalized coordinates are required to completely define its configuration. Hence, for an “ $n$ ” *degree of freedom system* in static equilibrium, “ $n$ ” *independent equilibrium equations* can be written.

#### Example

For the *simple pendulum* shown, the *torque*  $M$  required to hold the bar at some *constant* angle  $\theta$  can be calculated using the *principle of virtual work*. To do this, note that the weight force and the applied torque are the only *active forces and torques*. Using the angle  $\theta$  as the generalized coordinate, the principle of virtual work can be written as follows.

$$\begin{aligned} 0 = F_\theta &= (F_\theta)_W + (F_\theta)_M = \left( \tilde{W} \cdot \frac{\partial^R \tilde{v}_G}{\partial \dot{\theta}} \right) + \left( \tilde{M} \cdot \frac{\partial^R \tilde{\omega}_B}{\partial \dot{\theta}} \right) \\ &= \left( mg \tilde{j} \cdot \frac{L}{2} \left( \cos(\theta) \tilde{i} - \sin(\theta) \tilde{j} \right) \right) + (-M \tilde{k}) \cdot (-\tilde{k}) \\ \Rightarrow \boxed{M = mg \frac{L}{2} \sin(\theta)} \end{aligned}$$



**Contrast** this approach to *summing moments* on a free-body-diagram of the bar.

**Note:** The contribution of the *weight force* could also have been calculated using the *potential energy* function. Using the X-axis as the *horizontal datum* for the potential energy function, the *generalized force* associated with the weight force can be written as

$$(F_\theta)_W = -\frac{\partial}{\partial \theta} \left( -mg \frac{L}{2} \cos(\theta) \right) = -mg \frac{L}{2} \sin(\theta)$$