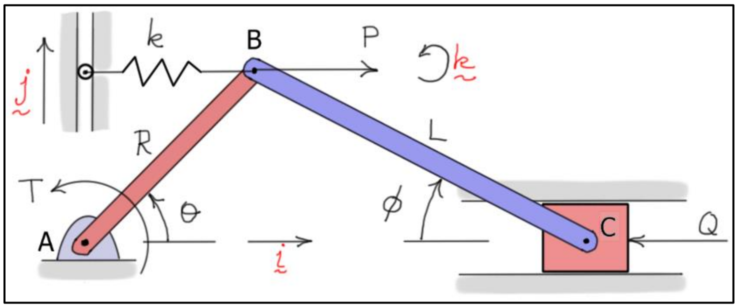
ME 5550 Intermediate Dynamics
Principle of Virtual Work – Example



The figure shows a ***slider crank mechanism*** with zero offset under the action of an ***external torque*** T acting on link AB, ***external forces*** P and Q acting at B and C, and a ***linear spring*** attached at B.

Problem: ***Given*** values for T and P and that the spring has unstretched length ℓ_u, ***find*** the force Q required to hold the mechanism in ***equilibrium*** at some angle θ.

Solution: (using θ as the ***generalized coordinate***)

Generalized Force (using some partial velocities from previous notes)

The ***generalized force*** associated with θ is: $\boxed{F_\theta = (F_\theta)_P + (F_\theta)_Q + (F_\theta)_{spring} + (F_\theta)_T}$

- $(F_\theta)_P = (P\underset{\sim}{i})\cdot\left(\dfrac{\partial \underset{\sim}{v}_B}{\partial\dot\theta}\right) = (P\underset{\sim}{i})\cdot\dfrac{\partial}{\partial\dot\theta}\left(R\dot\theta(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\right) = (P\underset{\sim}{i})\cdot\left(R(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\right) = \boxed{-PRS_\theta}$
- $(F_\theta)_{spring} = (-f_{sp}\underset{\sim}{i})\cdot\left(\dfrac{\partial \underset{\sim}{v}_B}{\partial\dot\theta}\right) = (-f_{sp}\underset{\sim}{i})\cdot\left(R(-S_\theta\underset{\sim}{i}+C_\theta\underset{\sim}{j})\right) = f_{sp}RS_\theta = \boxed{k(RC_\theta-\ell_u)RS_\theta}$
- $(F_\theta)_Q = (-Q\underset{\sim}{i})\cdot\left(\dfrac{\partial \underset{\sim}{v}_C}{\partial\dot\theta}\right) = (-Q\underset{\sim}{i})\cdot\left[-R\left(S_\theta + C_\theta S_\phi/C_\phi\right)\underset{\sim}{i}\right] = \boxed{QR\left(S_\theta + C_\theta S_\phi/C_\phi\right)}$
- $(F_\theta)_T = (T\underset{\sim}{k})\cdot\left(\dfrac{\partial \underset{\sim}{\omega}_{AB}}{\partial\dot\theta}\right) = T\underset{\sim}{k}\cdot\underset{\sim}{k} = \boxed{T}$

Principle of Virtual Work

Applying the ***principle of virtual work*** and solving for the force Q gives

$$F_\theta = 0 = -PRS_\theta + k(RC_\theta-\ell_u)RS_\theta + QR\left(S_\theta + C_\theta S_\phi/C_\phi\right) + T$$

$$\Rightarrow \boxed{Q = \frac{PRS_\theta - k(RC_\theta-\ell_u)RS_\theta - T}{R\left(S_\theta + C_\theta S_\phi/C_\phi\right)}}$$

Notes:

1. The ***pin forces*** at *A*, *B*, and *C* and the ***normal force*** at *C* are inactive (have no contribution).
2. The forces and torques that contribute to F_θ are ***active***.
3. There is only ***one equation*** associated with the one degree-of-freedom of the system.
4. The contribution of the spring could be calculated using ***potential energy***.