

# An Introduction to Three-Dimensional, Rigid Body Dynamics

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## Volume I – Kinematics: Summary of Contents

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### Unit 1 – Angular Velocity and Angular Acceleration: An Introduction

This unit introduces the concepts of *angular velocity* and *angular acceleration* vectors and shows *how to calculate* them for mechanical systems in which components are connected by simple revolute (pin) joints. These *concepts* will be *generalized* in Unit 5 to apply to systems with more *complex connecting joints*.

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### Unit 2 – Velocity and Acceleration Using Direct Differentiation

This unit introduces the concepts of *velocity* and *acceleration* vectors and shows *how to calculate* them using *direct differentiation*. This technique can be applied to complex mechanical systems; however, at this point the focus will remain on systems in which components are connected by simple revolute (pin) joints. This technique will be generalized in Unit 7 to apply to systems with more complex connecting joints.

Page Count	Examples	Suggested Exercises
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### Unit 3 – Relative Kinematics of Two Points Fixed on a Rigid Body

This unit continues the development of the concepts of *velocity* and *acceleration* vectors and shows *how to calculate* them using *formulae* that relate the motion of *two points fixed* on a rigid body. As before, this technique can be applied to complex mechanical systems; however, at this point the focus will remain on systems in which components are connected by simple revolute (pin) joints. This technique will be *generalized* in Unit 7 to apply to systems with more *complex connecting joints*.

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### Unit 4 – Kinematics of a Point Moving on a Rigid Body

This unit continues the development of the concepts of *velocity* and *acceleration* vectors and shows *how to calculate* them using formulae for points *moving on non-stationary rigid bodies*. As before, this technique can be applied to complex mechanical systems; however, at this point the focus will remain on systems in which components are connected by simple revolute (pin) joints. This technique will be *generalized* in Unit 7 to apply to systems with more *complex connecting joints*.

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## Unit 5 – Rigid Body Orientation, Orientation Angles, and Angular Velocity

In Unit 1 the concepts of *angular velocity* and *angular acceleration* were introduced, and examples were provided to illustrate how to calculate angular velocity and angular acceleration vectors for mechanical systems in which components are connected by simple revolute (pin) joints. These concepts will now be *generalized* and applied to bodies in three-dimensional motion irrespective of how they may be connected (if at all) to other bodies or to the ground. Specifically, this unit describes how to use *sequences of angles* to describe the *orientation* and *angular motion* of rigid bodies in three dimensions.

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## Unit 6 – Rigid Body Orientation, Orientation Parameters, and Angular Velocity

In Unit 5 the concept of using *orientation angles* to describe the orientation and angular velocity of a rigid body was presented. It was noted, however, that *all orientation angle sequences* display a *singularity* at some orientation and that this can cause problems for computer programs that use them. This unit shows how to use the *orientation parameters* known as *Euler parameters* to remedy this situation.

Some *limited proofs* of properties associated with Euler parameters are provided in the *Addendum* to this unit. A detailed understanding of those proofs, however, is *not necessary* when applying these concepts.

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## Unit 7 – Application of Concepts to Systems with Complex Interconnecting Joints

The *basic kinematical concepts* for mechanical systems were presented in Units 1 through 4. The concepts were applied to systems in which the *relative orientation changes* between adjoining bodies were described by a *single rotation* - simple revolute joints. In Units 5 and 6 concepts were presented for describing the relative orientation changes between *any two adjoining bodies* by using a set of up to three *orientation angles* or a set of four *Euler parameters*. This unit *applies* the concepts from Units 1 through 6 to analyze the kinematics of systems with *more complex connecting joints*. The major differences in applying the kinematical concepts of Units 1 through 4 are in the calculations of orientation, angular velocity and angular acceleration.

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## Unit 8 – Introduction to Systems with Closed Kinematic Chains

This unit provides an *introduction* to systems that have *closed kinematic chains*. Previous units focused on how to *constrain* the motion between *adjoining bodies* that form *open-tree* or *open-chain* systems. In systems that have *closed kinematic chains* (or *closed loops*), two points within a chain of rigidly interconnected bodies have *specified* or *constrained motion* (relative to the ground, for example). This unit will discuss how to *include additional kinematic equations* (constraints) that apply when a system has such chains. The coverage is *not meant to be comprehensive* but should give the reader a solid introduction to the *additional complexities* involved with connecting bodies in three dimensional systems.

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## Unit 9 – Introduction to Systems with Rolling Constraints

This unit provides an *introduction* to systems having *rolling constraints*. When the surface of a rigid body is said to be “*rolling*” on another rigid surface (possibly of a second rigid body), it is assumed there is *no slippage* between the two surfaces. In this case, even though the two surfaces are *moving relative to each other*, the *velocities* of the *contact points* on each of the surfaces must be the *same*. This unit discusses rolling constraints for two common situations – bodies having a *single contact point* and bodies that have a *line of contact points*. Applications include ball bearings, roller bearings and gearing systems. Two- dimensional systems with rolling constraints are also included to compare and contrast with three-dimensional systems. The coverage is *not meant to be comprehensive* but should give the reader a solid introduction to the *additional complexities* involved with rolling bodies.

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## Unit 10 – Introduction to Modeling Mechanical System Kinematics using MATLAB<sup>®</sup> Scripts, Simulink<sup>®</sup> and SimMechanics<sup>®</sup>

This unit provides an introduction to *modeling* mechanical system kinematics using *MATLAB scripts*, Simulink *models*, and *SimMechanics models*. MATLAB scripts are *text-based programs* written in the MATLAB programming language. Simulink models are *block-diagram-based programs* that run in the MATLAB environment. SimMechanics models are *block-diagram-based, multibody dynamics programs* that run in the MATLAB/Simulink environment. MATLAB scripts can be used *alone or in conjunction with* Simulink and SimMechanics models.

Developing models using MATLAB scripts alone requires an analyst to *write* or *computationally develop* all the necessary equations to model the system. Many solution algorithms built into MATLAB and its toolboxes can be used to *solve* the equations. This is a *detailed process* and may be too time-intensive for the average analyst to master. Alternatively, Simulink can be used to *speed-up model development* by allowing the solution process to be programmed in the form of block diagrams. Like MATLAB, Simulink has many built-in algorithms to lighten the analyst’s load. Because these two approaches give the analyst *total control*

*of all modeling and solution details*, they provide the *most flexible* modeling environment. However, the amount of time and effort required to develop models using these approaches *increases* rapidly as the system complexity increases.

SimMechanics models are Simulink models that contain *special blocks* for modeling *multibody dynamics*. These blocks eliminate the need for analysts to develop a set of equations of motion of their own. The analyst builds a block diagram model of the system, and SimMechanics *develops and solves* the equations of motion. These models require *less time and knowledge* to develop, but they are also less flexible than those discussed above in that the analyst *does not have access* to the detailed equations or the solution process.

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