Introductory Control Systems
Exercises #6 – Transfer Functions from Closed Loop Equations

Sketch a ***closed-loop block diagram*** for each of the following sets of closed-loop equations. Identify all ***signals*** and ***transfer functions***. Then, find the indicated closed-loop transfer function.

1. ***Speed control*** of a car in the presence of a ***disturbance*** force is described by the boxed equations. The ***desired speed*** is $r(t)$, the ***actual speed*** is $v(t)$, the ***speed error*** is $e(t)$, the ***driving force*** on the car is $f_a(t)$, the ***disturbance force*** on the car is $f_d(t)$, and the ***net force*** on the car is $f_{net}(t)$. Find $\frac{V}{R}(s)$.

$$\frac{dv}{dt}+3v = f_{net}(t)$$
$$f_{net}(t) = f_a(t) - f_d(t)$$
$$f_a(t) = Ke(t)$$
$$e(t) = r(t) - v(t)$$

2. The boxed equations describe ***position control*** of a spring-mass system using ***proportional*** (P) control. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $y(t)$, the ***position error*** is $e(t)$, and the ***actuator force*** applied to the mass is $f(t)$. Find $\frac{Y}{R}(s)$.

$$e(t) = r(t) - y(t)$$
$$f(t) = K\, e(t)$$
$$\ddot{y} + 4y = f(t)$$

3. The boxed equations describe ***position control*** of a spring-mass-damper system using ***proportional-derivative*** (PD) control. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $x(t)$, the ***position error*** is $e(t)$, and the ***actuating force*** is $f(t)$. Find $\frac{X}{R}(s)$.

$$e(t) = r(t) - x(t)$$
$$f(t) = \dot{e}(t) + 5e(t)$$
$$\ddot{x} + 6\dot{x} + 20x = f(t)$$

4. ***Position control*** of a hydraulic actuator using ***proportional-integral*** (PI) control is described by the boxed equations. The desired position is $r(t)$, the actual position is $x(t)$, the position error is $e(t)$, and the actuating forces is $f(t)$. Find the ***transfer function*** $\frac{X}{R}(s)$.

$$e(t) = r(t) - x(t)$$
$$f(t) = e(t) + 3\int_0^t e(t)dt$$
$$\ddot{x} + 5\dot{x} = 2f(t)$$

5. The ***speed control*** of a car using proportional-integral (PI) control is described by the boxed equations. The ***desired speed*** is $r(t)$, the ***actual speed*** $v(t)$, the ***speed error*** is $e(t)$, and the ***driving force*** is $f(t)$. Find $\frac{V}{R}(s)$.

$$m\dot{v} + cv = f(t)$$
$$e(t) = r(t) - v(t)$$
$$f(t) = K_1 e(t) + K_2\int_0^t e(t)dt$$

6. The boxed equations describe ***speed control*** of a rotating mass system using proportional control. The ***desired rotational speed*** is $r(t)$, the ***actual rotational speed*** is $\omega(t)$, the ***rotational speed error*** is $e(t)$, the ***input voltage*** to the motor drive actuator is $v(t)$, and the ***actuator torque*** applied to the mass is $M(t)$. Find $\frac{\omega}{R}(s)$.

$$e(t) = r(t) - \omega(t)$$
$$v(t) = K e(t)$$
$$\dot{M} + 8M = v(t)$$
$$\dot{\omega} + 7\omega = 3M(t)$$

7. The boxed equations describe ***position control*** of a spring-mass system using proportional control with a linear actuator. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $y(t)$, the ***position error*** is $e(t)$, and the ***actuator force*** applied to the mass is $f(t)$. Find $\frac{Y}{R}(s)$.

$$\boxed{\begin{aligned} &e(t) = r(t) - y(t) \\ &v(t) = 10\, e(t) \\ &\dot{f} + 3f = v(t) \\ &\ddot{y} + 9y = f(t) \end{aligned}}$$