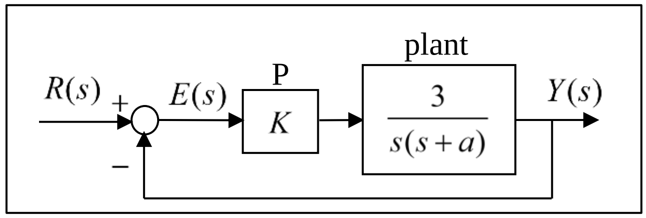
Introductory Control Systems
Exercises #8 – Closed-Loop, Second-Order System Step Response

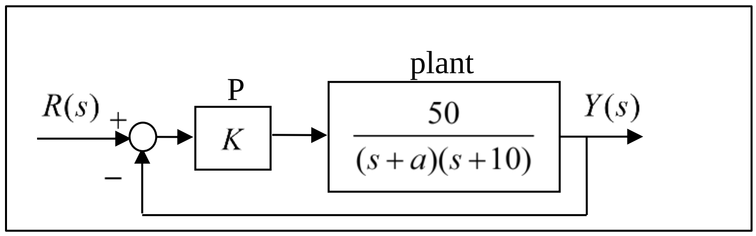
In the answers that follow, "Fig. 5.8" and "Fig. 5.13" refer to figures 5.8 and 5.13 in Dorf & Bishop, *Modern Control Systems*, 11th Ed., Pearson/Prentice-Hall, 2008. The estimates are based on reading values from those graphs.

1. A ***proportional*** ("P") controller is used to control a 2nd order plant as shown. The system input is $R(s)$ and the system output is $Y(s)$. Given $K=12$ and $a=3$, estimate the ***percent overshoot*** $(\%OS)$ and the ***2% settling time*** (T_s) of the closed-loop system to a ***unit step input***.



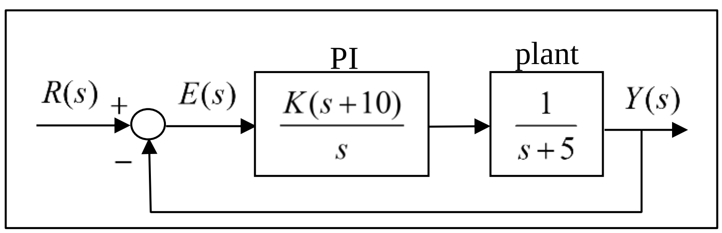
Answers: $\frac{Y}{R}(s)=\frac{36}{s^2+3s+36}$, (2nd order, case 1), $\%OS \approx 45\%$ (Fig 5.8), $T_s \approx 2.67$ (sec)

2. ***Proportional*** ("P") control is used to control a 2nd order plant as shown. The system has input $R(s)$ and output $Y(s)$. Given $K=3$ and $a=5$, estimate the ***percent overshoot*** $(\%OS)$ and the ***2% settling time*** (T_s) of the closed-loop system to a ***unit step input***.



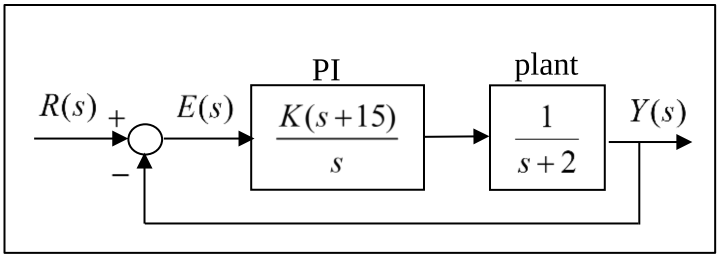
Answers: $\frac{Y}{R}(s)=\frac{150}{s^2+15s+200}$, (2nd order, case 1), $\%OS \approx 15\%$ (Fig 5.8), $T_s \approx 0.533$ (sec)

3. The diagram shows a closed-loop system with ***proportional-integral*** (PI) control of a 1st order plant. The system has input $R(s)$ and output $Y(s)$. Given that $K=15$, estimate the ***percent overshoot*** $(\%OS)$ and the ***2% settling time*** (T_s) of the closed loop system to a ***unit step input***.



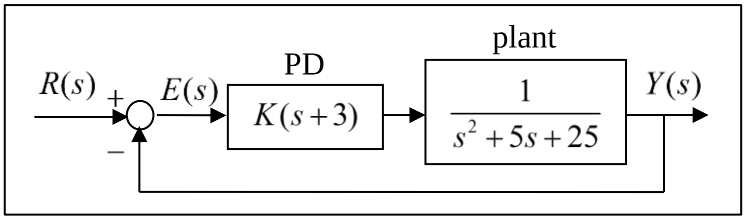
Answers: $\frac{Y}{R}(s)=\frac{15(s+10)}{s^2+20s+150}$; $\%OS \approx 9\%$ (Fig. 5.13); $T_s \approx 0.4$ (sec)

4. The diagram shows a closed-loop system with ***proportional-integral*** (PI) control. The system has ***input*** $R(s)$ and ***output*** $Y(s)$. Given $K = 34$, estimate the ***percent overshoot*** $(\%OS)$ and the ***2% settling time*** (T_s) of the system to a ***unit step input***.



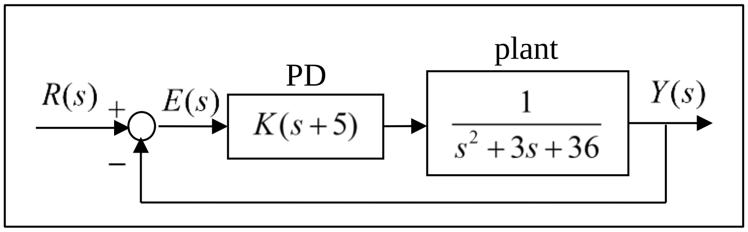
Answers: $\frac{Y}{R}(s) = \frac{34(s+15)}{s^2+36s+510}$, (2nd order, case 2), $\%OS \approx 13-14\%$ (Fig 5.13), $T_s \approx 0.222$ (sec)

5. A ***proportional-derivative*** ("PD") controller is used to control a ***2nd order*** plant as shown. The system has input $R(s)$ and output $Y(s)$. Assuming $K = 3.25$, estimate the percent overshoot $(\%OS)$ and settling time (T_s) of the closed-loop system.



Answers: $\frac{Y}{R}(s) = \frac{3.25(s+3)}{s^2+8.25s+34.75}$, (2nd order, case 2), $\%OS \approx 40\%$ (Fig 5.13), $T_s \approx 0.97$ (sec)

6. A ***proportional-derivative*** ("PD") controller is used to control a ***2nd order plant*** as shown. The system has input $R(s)$ and output $Y(s)$. Assuming $K = 7.17$, estimate the percent overshoot $(\%OS)$ and settling time (T_s) of the closed-loop system.



Answers: $\frac{Y}{R}(s) = \frac{7.17(s+5)}{s^2+10.17s+71.85}$, $\%OS \approx 40\%$ (Fig. 5.13), $T_s \approx 0.787$ (sec)