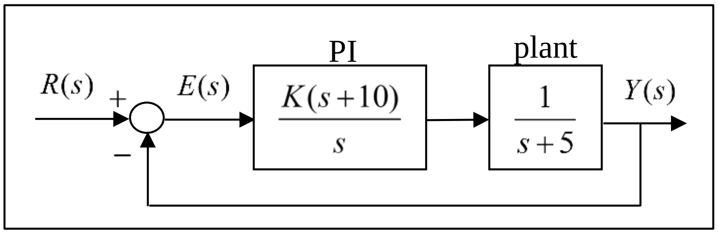
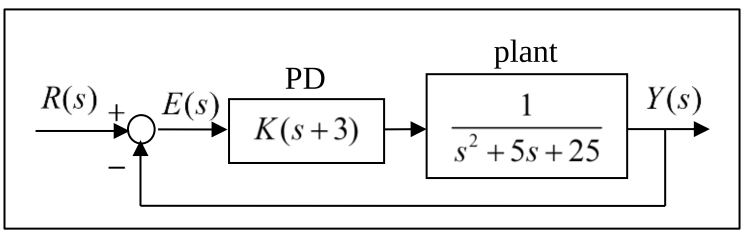
Introductory Control Systems
Exercises #10 – Steady-State Error

1. A ***proportional-integral*** ("PI") controller is used to control a 1st order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\frac{E}{R}(s)$ the ***error transfer function***, and then find the range of values for the parameter K so the system has a ***steady-state error*** e_{ss} less than 0.01 for a ***unit ramp*** input $\left(R(s)=1/s^2\right)$. Assume $K>0$.



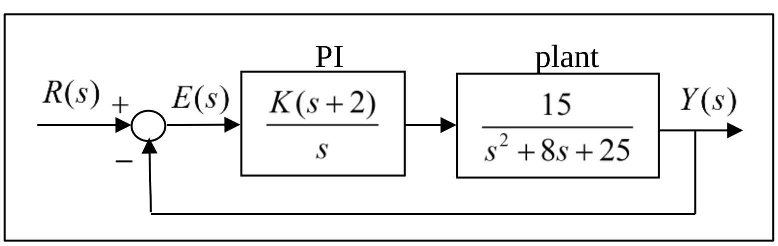
Answers: $\boxed{\frac{E}{R}(s)=\frac{s(s+5)}{s(s+5)+K(s+10)}}$; $K>50$

2. A ***proportional-derivative*** ("PD") controller is used to control a 2nd order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\frac{E}{R}(s)$ the ***error transfer function***, and find e_{ss} the ***steady-state error*** associated with a ***unit step*** input in terms of the parameter K.



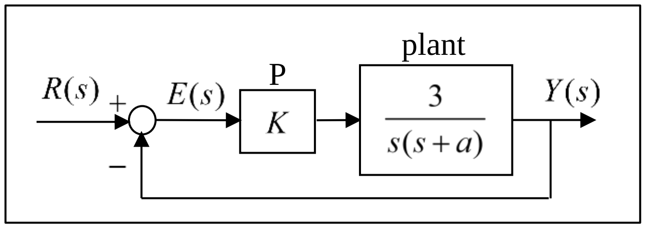
Answers: $\boxed{\frac{E}{R}(s)=\frac{s^2+5s+25}{s^2+5s+25+K(s+3)}}$; $\boxed{e_{ss}=\frac{25}{25+3K}}$

3. A ***proportional-integral*** ("PI") controller is used to control a 2nd order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\frac{E}{R}(s)$ the ***error transfer function***, and then find the range of values for the parameter K so the system has a steady-state error less than 0.1 to a ***unit ramp*** input $\left(R(s)=1/s^2\right)$. Assume $K>0$.



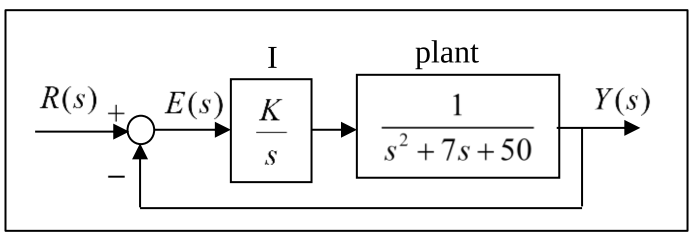
Answers: $\boxed{\dfrac{E}{R}(s)=\dfrac{s(s^2+8s+25)}{s(s^2+8s+25)+15K(s+2)}}$; $\boxed{K>8.33}$

4. A ***proportional*** ("P") controller is used to control a 2nd order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\dfrac{E}{R}(s)$ the ***error transfer function***, and then find e_{ss} the ***steady-state error*** associated with a ***unit ramp*** input in terms of the parameters a and K.



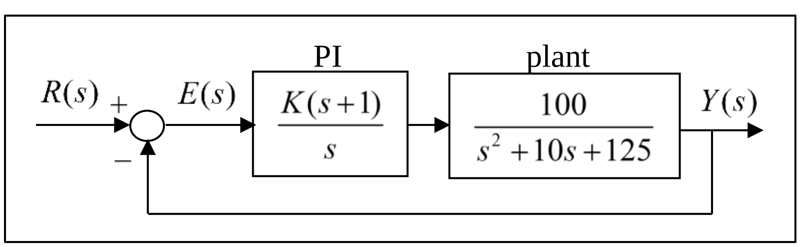
Answers: $\boxed{\dfrac{E}{R}(s)=\dfrac{s(s+a)}{s^2+as+3K}}$, $\boxed{e_{ss}=\dfrac{a}{3K}}$

5. An ***integral*** ("I") controller is used to control a 2nd order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\dfrac{E}{R}(s)$ the ***error transfer function***, and then find the range of values for the gain K so e_{ss} the steady-state error due to a ***unit ramp*** input is less than 1.0. Assume $K>0$.



Answers: $\boxed{\dfrac{E}{R}(s)=\dfrac{s(s^2+7s+50)}{s^3+7s^2+50s+K}}$; $\boxed{K>50}$

6. A ***proportional-integral*** ("PI") controller is used to control a 2nd order plant as shown. The system has input $R(s)$, output $Y(s)$, and error $E(s)$. Find $\dfrac{E}{R}(s)$ the ***error transfer function***, and then find the range of values for the parameter K so the steady-state error to a ***unit ramp*** input is less than 0.1. Assume $K>0$.



Answers: $\boxed{\dfrac{E}{R}(s)=\dfrac{s(s^2+10s+125)}{s(s^2+10s+125)+100K(s+1)}}$; $\boxed{K>12.5}$