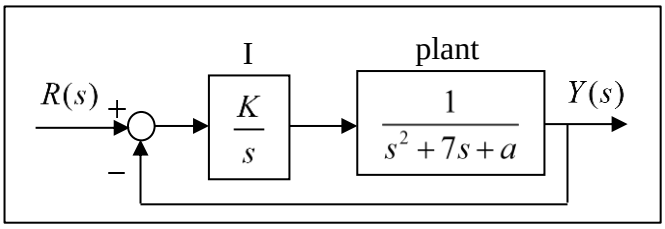
Introductory Control Systems
Exercises #11 – Performance Indices

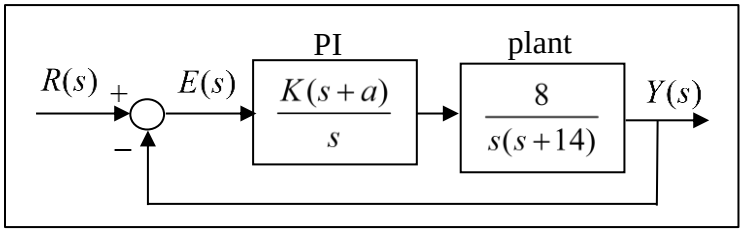
The results given below for ITAE optimal response to step and ramp input were found using Tables 5.6 and 5.7 in R.C. Dorf and R.H. Bishop, *Modern Control Systems*, 11th Ed., Pearson Prentice Hall, 2008.

1. An ***integral*** ("I") controller is used to control a 2nd order plant as shown. The system has input $R(s)$ and output $Y(s)$. Find values for the parameters K and a so the closed-loop system has ***ITAE optimal response*** to a ***step input***.



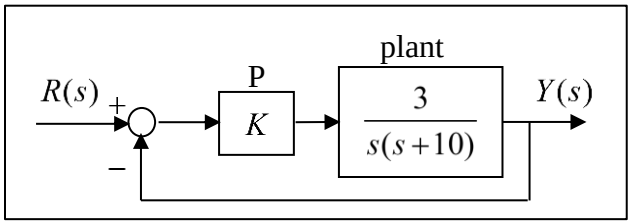
Answers: $\boxed{\frac{Y}{R}(s)=\frac{K}{s^3+7s^2+a\,s+K}}$; $\boxed{a\approx 34.4}$; $\boxed{K\approx 64}$

2. A proportional/integral ("PI") controller is used to control a 2nd order plant as shown. The system has input $R(s)$ and output $Y(s)$. Find values for the parameters K and a so the closed-loop system has ***ITAE optimal response*** to a ***ramp input***.



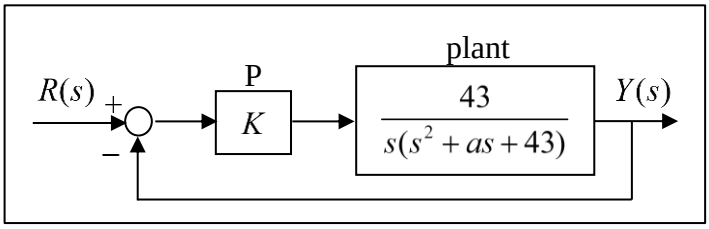
Answers: $\boxed{\frac{Y}{R}(s)=\frac{8K(s+a)}{s^3+14s^2+8Ks+8aK}}$; $\boxed{K=26}$; $\boxed{a\approx 2.46}$

3. A ***proportional*** ("P") controller is used to control a 2nd order plant as shown. The system has input $R(s)$ and output $Y(s)$. Find the value of K so that the closed-loop system has ***ITAE optimal response*** to a ***step input***.



Answers: $\boxed{\frac{Y}{R}(s)=\frac{3K}{s^2+10s+3K}}$; $\boxed{K\approx 17.0}$

4. A ***proportional*** ("P") controller is used to control a 3rd order plant as shown in the diagram. The system has input $R(s)$ and output $Y(s)$. Find the values of the parameters a and K that will give the closed-loop system an ***ITAE optimal response*** to a ***step input***.



Answers: $\boxed{\frac{Y(s)}{R(s)} = \frac{43K}{s^3 + as^2 + 43s + 43K}}$; $\boxed{a \approx 7.83}$; $\boxed{K \approx 2.08}$