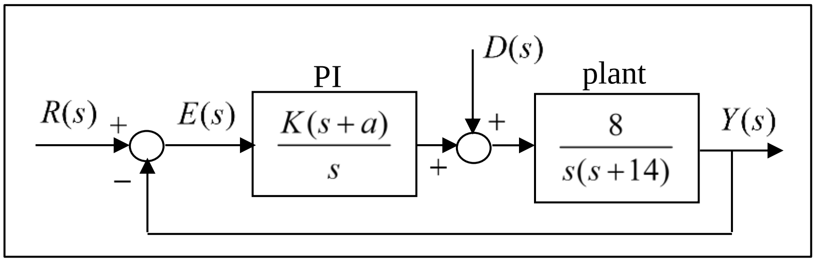
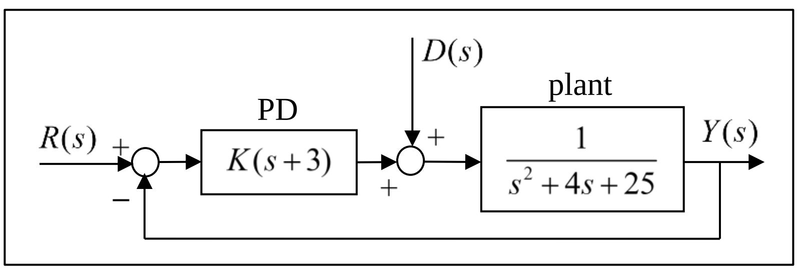
Introductory Control Systems
Exercises #12 – Disturbance Response

1. A ***proportional/integral*** ("PI") controller is used to control a 2nd order plant as shown. The system has two input signals ($R(s)$ and $D(s)$) and one output signal ($Y(s)$). Find the ***disturbance transfer function*** $\frac{Y}{D}(s)$. Then, find the range of the product "Ka" so $(y_{ss})_D$ the ***steady state system response*** to a ***unit ramp disturbance input*** $(D(s)=1/s^2)$ is less than 0.01.



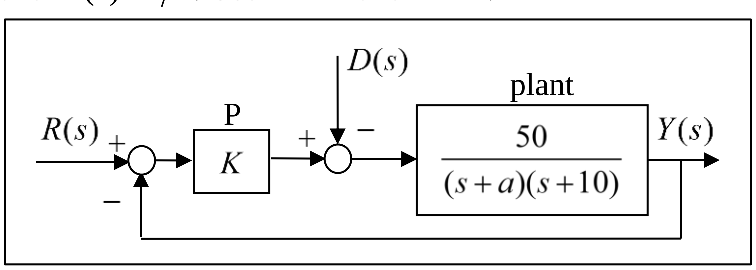
Answers: $\boxed{\frac{Y}{D}(s)=\frac{8s}{s^3+14s^2+8Ks+8aK}}$; $\boxed{(K\,a)>100}$

2. A ***proportional-derivative*** ("PD") controller is used to control a 2nd order plant as shown. The system input is $R(s)$, the disturbance input is $D(s)$, and the output is $Y(s)$. Find $\frac{Y}{D}(s)$ the ***disturbance transfer function*** and find $(y_{ss})_D$ the ***steady-state*** response associated with a ***unit step*** disturbance input.



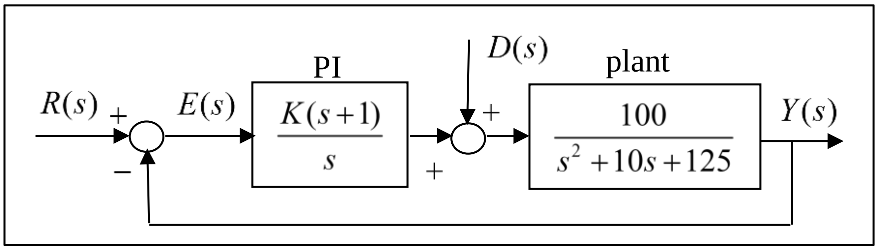
Answers: $\boxed{\frac{Y}{D}(s)=\frac{1}{s^2+(4+K)s+(25+3K)}}$; $\boxed{(y_{ss})_D=\frac{1}{25+3K}}$

3. ***Proportional*** ("P") control is used to control a 2nd order plant as shown. The system has input signals $R(s)$ and $D(s)$ and output $Y(s)$. Find y_{ss} the final (steady-state) value of $y(t)$ associated with ***simultaneous unit step inputs*** $R(s)=1/s$ and $D(s)=1/s$. Use $K=3$ and $a=5$.



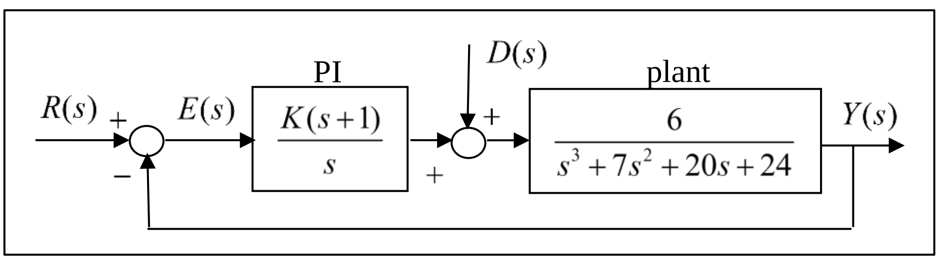
Answers: $\boxed{\dfrac{Y}{R}(s)=\dfrac{150}{s^2+15s+200}}$; $\boxed{\dfrac{Y}{D}(s)=\dfrac{-50}{s^2+15s+200}}$; $\boxed{y_{ss}=0.5}$

4. The diagram shows a closed-loop system with ***proportional-integral*** ("PI") control. The system has ***command input*** $R(s)$, ***disturbance input*** $D(s)$, and ***output*** $Y(s)$. Find $\dfrac{Y}{D}(s)$ the ***disturbance transfer function***, and then find the range for the parameter K so that the system output is less than 0.1 to a ***unit ramp disturbance*** input ($D(s)=1/s^2$). Assume $K>0$.



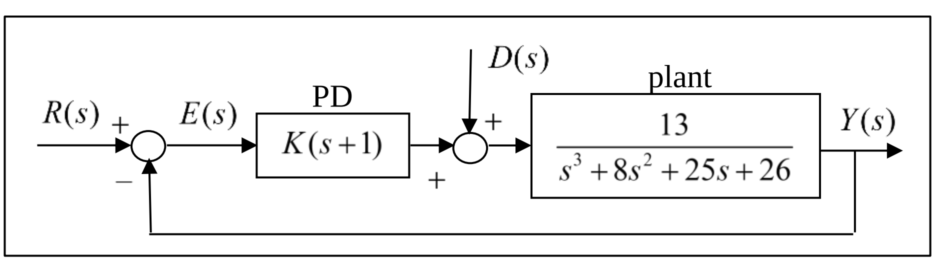
Answers: $\boxed{\dfrac{Y}{D}(s)=\dfrac{100s}{s(s^2+10s+125)+100K(s+1)}}$, $\boxed{K>10}$

5. The diagram shows a closed-loop system with ***proportional-integral*** ("PI") control. The system has command input $R(s)$, disturbance input $D(s)$, and output $Y(s)$. Find $\dfrac{Y}{D}(s)$ the disturbance transfer function, and then find the range for the parameter K so that the system output is less than 0.1 to a ***unit ramp disturbance*** input ($D(s)=1/s^2$). Assume $K>0$.



Answers: $\boxed{\dfrac{Y(s)}{D(s)}=\dfrac{6s}{s(s^3+7s^2+20s+24)+6K(s+1)}}$; $\boxed{K>10}$

6. The diagram shows a closed-loop system with ***proportional-derivative*** ("PD") control. The system has ***command input*** $R(s)$, ***disturbance input*** $D(s)$, and ***output*** $Y(s)$. Find y_{ss} the steady state response of the system to ***simultaneous command and disturbance step inputs*** ($R(s)=1/s$ and $D(s)=1/s$). Assume the parameter $K=20$.



Answers: $\boxed{\dfrac{Y}{R}(s)=\dfrac{13K(s+1)}{s^3+8s^2+25s+26+13K(s+1)}}$; $\boxed{\dfrac{Y}{D}(s)=\dfrac{13}{s^3+8s^2+25s+26+13K(s+1)}}$; $\boxed{y_{ss}\approx 0.955}$