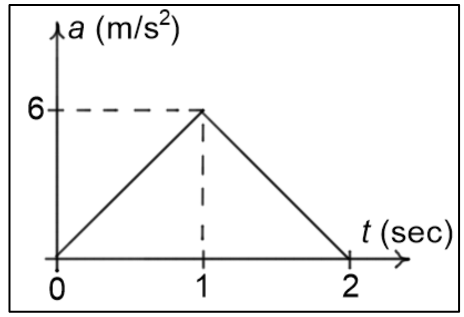
Elementary Dynamics
Exercises #1 – Rectilinear Motion Profiles

1. The cart travels along the horizontal path. It has an acceleration as shown in the diagram. The initial position and velocity of the cart are $s_0 = 0$ (ft) and $v_0 = 2$ (ft/sec). Find the following: a) $v(t)$ the velocity of the cart for $0 \le t \le 1$ (sec), b) $s(t)$ the position of the cart for $0 \le t \le 1$ (sec), c) $v(t)$ the velocity of the cart for $1 \le t \le 2$ (sec), and d) $s(t)$ the position of the cart for $1 \le t \le 2$ (sec).



Answers:

a) $v(t) = 2 + 3t^2$ (ft/sec) $0 \le t \le 1$ (sec); b) $s(t) = 2t + t^3$ (ft) $0 \le t \le 1$ (sec)
c) $v(t) = -4 + 12t - 3t^2$ (ft/sec) $1 \le t \le 2$ (sec); d) $s(t) = 2 - 4t + 6t^2 - t^3$ (ft) $1 \le t \le 2$ (sec)

2. A car travels along a horizontal path with an ***acceleration*** $a(s) = 20 - 3s$ (m/s^2), where s is in meters. The ***initial velocity*** of the car is $v(0) = 30$ (m/s). Find the following: a) $v(s)$ the ***velocity*** of the car as a function of position s, b) the ***velocity*** of the car after it travels 10 (m), and c) d the ***distance*** required for the car to stop.

Answers:

a) $v(s) = \sqrt{900 + 40s - 3s^2}$ (m/s); b) $v(10) = 31.6$ (m/s); c) $d = 25.2$ (m)

3. A car travels along a horizontal path with an ***acceleration*** $a(v) = -\frac{1}{4}v^2$ (m/s^2), where v is in meters per second. The ***initial velocity*** of the car is $v(0) = 16$ (m/s). Find $v(t)$ the ***velocity*** of the car as a function of time.

Answer: $v(t) = \dfrac{16}{4t+1}$

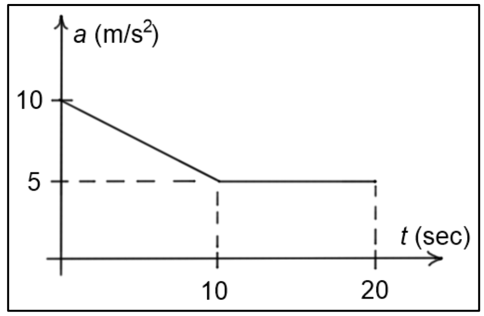
4. Accounting for aerodynamic drag, the acceleration of a car is $a(v) = 30 - \frac{v^2}{400}$ (ft/s^2). If the car starts from rest, find the displacement of the car when its velocity is $v = 50$ (ft/s).

Answer: $s = 46.7$ (ft)

5. A car travels along a horizontal path with an ***acceleration*** $a(v) = -3v^{\frac{2}{3}}$ (m/s^2), where v is in ***meters per second***. Find $v(s)$ the ***velocity*** of the car as a function of s (the distance traveled). The ***initial velocity*** of the car is 27 (m/s) when $s = 0$.

Answer: $v(s) = (81 - 4s)^{\frac{3}{4}}$ (m/s)

6. A cart travels along a horizontal path with acceleration $a(t) = 10 - \frac{1}{2}t$ (m/s^2) for $0 \le t \le 10$ (s) and ***constant*** acceleration $a = 5$ (m/s^2) for $10 \le t \le 20$ (sec). The initial position of the cart is $s(0) = 0$ (m), and its initial velocity is $v(0) = 3$ (m/s). Find the following: a) $v(t)$ the velocity of the cart as a function of time for $0 \le t \le 10$ (s), b) $s(t)$ the displacement of the cart as a function of time for $0 \le t \le 10$ (s), c) $v(t)$ the velocity of the cart as a function of time for $10 \le t \le 20$ (s), and d) $s(t)$ the displacement of the cart as a function of time for $10 \le t \le 20$ (s).

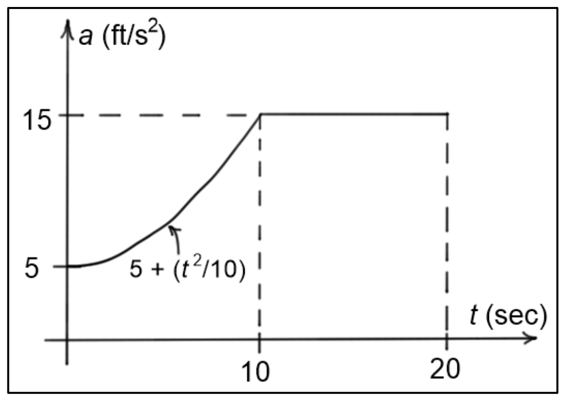


Answers:

a) $v(t) = 3 + 10t - \frac{1}{4}t^2$ (m/s) $\quad 0 \le t \le 10$ (s); b) $s(t) = 3t + 5t^2 - \frac{1}{12}t^3$ (m) $\quad 0 \le t \le 10$ (s)

c) $v(t) = 5t + 28$ (m/s) $\quad 10 \le t \le 20$ (s); d) $s(t) = \frac{5}{2}t^2 + 28t - 83.3$ (m) $\quad 10 \le t \le 20$ (s)

7. A car travels along a horizontal path and has ***acceleration*** $a = 5 + \frac{1}{10}t^2$ (ft/s^2) for $0 \le t \le 10$ (sec) and ***constant*** acceleration $a = 15$ (ft/s^2) for $10 \le t \le 20$ (sec). The car starts from ***rest*** at $s = 0$ when $t = 0$. Find the following: a) $v(t)$ the velocity of the car as a function of time for $0 \le t \le 10$ (sec), b) $s(t)$ the displacement of the car as a function of time for $0 \le t \le 10$ (sec), c) $v(t)$ the velocity of the car as a function of time for $10 \le t \le 20$ (sec), and d) $s(t)$ the displacement of the car as a function of time for $10 \le t \le 20$ (sec).

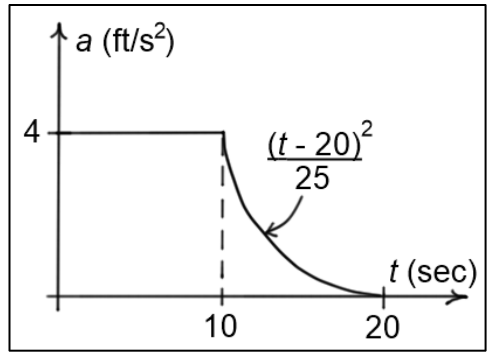


Answers:

a) $v(t) = 5t + \frac{t^3}{30}$ (ft/s) $\quad 0 \le t \le 10$ (sec); b) $s(t) = \frac{5}{2}t^2 + \frac{t^4}{120}$ (ft) $\quad 0 \le t \le 10$ (sec)

c) $v(t) = 15t - 66.7$ (ft/s) $\quad 10 \le t \le 20$ (sec); d) $s(t) = 7.5t^2 - 66.7t + 250.0$ (ft) $\quad 10 \le t \le 20$ (sec)

8. A car travels along a horizontal path and has ***constant*** acceleration of $a = 4\ (\text{ft/s}^2)$ for $0 \le t \le 10\ (\text{sec})$ and acceleration of $a(t) = \frac{1}{25}(t-20)^2\ (\text{ft/s}^2)$ for $10 \le t \le 20\ (\text{sec})$. The car starts from ***rest*** at $s = 0$. Find the following: a) $v(t)$ the velocity of the car as a function of time for $0 \le t \le 10\ (\text{sec})$, b) $s(t)$ the displacement of the car as a function of time for $0 \le t \le 10\ (\text{sec})$, c) $v(t)$ the velocity of the car as a function of time for $10 \le t \le 20\ (\text{sec})$, and d) $s(t)$ the displacement of the car as a function of time for $10 \le t \le 20\ (\text{sec})$.



Answers:

a) $v(t) = 4t\ (\text{ft/s}) \qquad 0 \le t \le 10\ (\text{sec})$; b) $s(t) = 2t^2\ (\text{ft}) \qquad 0 \le t \le 10\ (\text{sec})$

c) $v(t) = \frac{1}{75}(t-20)^3 + 53.3\ (\text{ft/s}) \qquad 10 \le t \le 20\ (\text{sec})$

d) $s(t) = \frac{1}{300}(t-20)^4 + 53.3t - 367\ (\text{ft}) \qquad 10 \le t \le 20\ (\text{sec})$