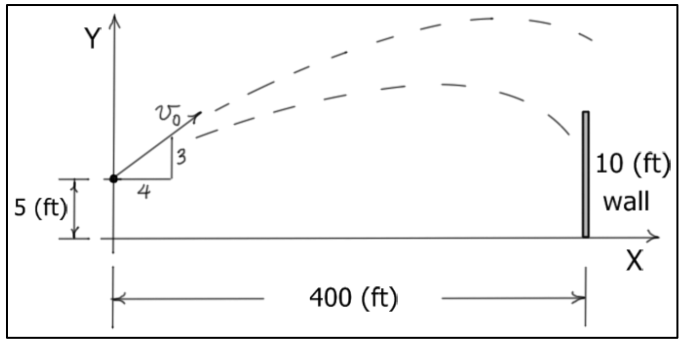
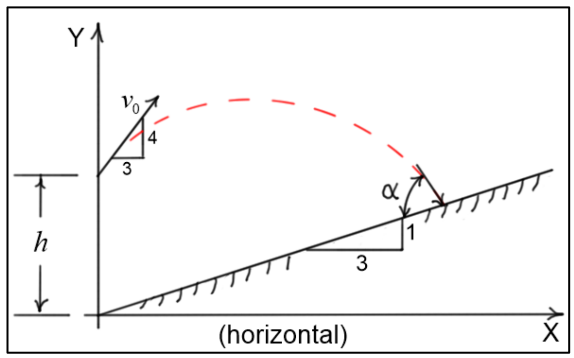
Elementary Dynamics
Exercises #2 – Curvilinear Motion: Rectangular Coordinates

1. A baseball is hit with an ***initial speed*** $v_0 = 120$ (ft/sec) at the angle shown. Complete the following: a) Find expressions for $x(t)$ and $y(t)$ the ***horizontal*** and ***vertical*** coordinates of the baseball ***at any time*** t. b) Does the ball hit the wall or go over? c) What ***initial speed*** v_0 is required to ***hit*** the ***top of the wall*** at $(x, y) = (400, 10)$ (ft)?



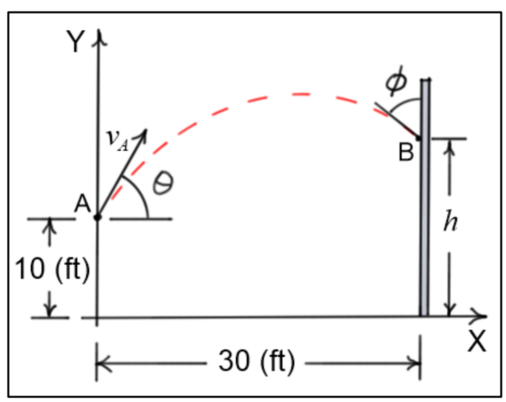
Answers:
   a) $x(t) = 96t$ and $y(t) = 5 + 72t - 16.1t^2$
   b) time to reach wall, $t^* = 4.1\overline{6}$ (sec); $y(t^*) \approx 25.5$ (ft) > 10 (ft); goes over wall
   c) $v_0 \approx 116.808 \approx 117$ (ft/s) (time to reach wall, $t^* \approx 4.28$ (sec))

2. A baseball is thrown from a height of $h = 7$ (m) at an initial speed $v_0 = 10$ (m/s) as shown. Complete the following: a) Find $y = y(x)$ the equation of the parabolic path of the ball. b) Find the x^* and y^* coordinates of the location where the ball strikes the slope. c) How long does it take for the ball to strike the slope? d) At what angle α does the ball strike the slope?



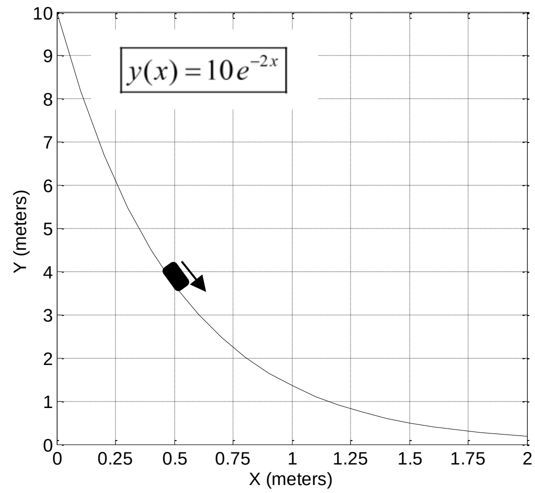
Answers:
   a) $y(x) = 7 + \frac{4}{3}x - \left(\frac{9.81}{72}\right)x^2$ (m) '; b) $\left(x^*, y^*\right) = (11.72, 3.907)$ (m); c) $t = 1.954$ (sec); d) $\alpha = 80.19$ (deg)

3. A ball is thrown from a position 10 feet above the ground with and initial velocity of $v_A = 40$ (ft/sec) at an angle of $\theta = 60$ (deg) relative to the horizontal. The wall is 30 feet away from where the ball is thrown. Find: a) $x(t)$ and $y(t)$ the X and Y coordinates of the ball as functions of time; b) h the height above the ground where the ball hits the wall; c) ϕ the angle that the ball strikes the wall; d) $\dot{v}_B$ the rate of change of speed of the ball just before it strikes the wall at B.



Answers:
   a) $x(t) = 20t$ (ft) and $y(t) = 10 + 34.64t - 16.1t^2$ (ft); b) $h = 25.7$ (ft); c) $\phi = 55.7$ (deg)
   d) $\dot{v}_B = 18.1$ (ft/s^2)

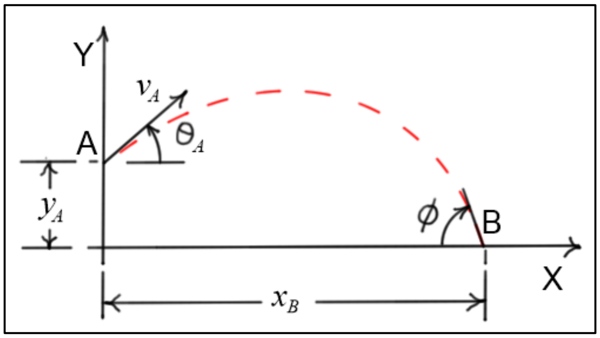
4. A sled is traveling down a hill which can be approximated by the function $y(x) = 10e^{-2x}$. When the sled is at $x = 0.5$ (m), its components of velocity and acceleration in the X-direction are $\dot{x} = 3$ (m/s) and $\ddot{x} = 5$ (m/s^2). Find: a) $|\underset{\sim}{v}|$ the magnitude of the velocity of the sled at $x = 0.5$ (m); b) θ the angle of the tangent line relative to the X-axis at $x = 0.5$ (m); c) $|\underset{\sim}{a}|$ the magnitude of the acceleration of the sled at $x = 0.5$ (m).



Answers:

a) $|\underset{\sim}{v}| = 22.3$ (m/s); b) $\theta = -82.3$ (deg); c) $|\underset{\sim}{a}| = 95.8$ (m/s^2)

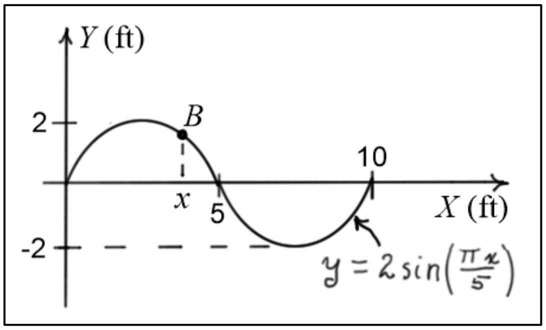
5. A ***projectile*** is fired from a position $y_A = 20$ (ft) ***above*** the ground and ***hits*** the ground at $x_B = 200$ (ft). The ***time*** required for the ball to go from A to B is $\hat{t} = 5$ (sec). Find: a) v_A the ***initial velocity*** of the projectile; b) θ_A the ***angle*** at which it is fired; c) ϕ the ***angle*** at which the projectile ***strikes the ground***.



Answers:

a) $v_A = 86.3$ (ft/s); b) $\theta_A = 62.4$ (deg); c) $\phi = 64.7$ (deg)

6. The bead B is moving along a wire which has a sinusoidal shape given by the function $y = 2\sin\left(\frac{\pi x}{5}\right)$. The component of its velocity in the X direction is a ***constant*** $v_x = \dot{x} = 3$ (ft/s). Find v_B the magnitude of the velocity of B when $x = 3$ (ft).



Answer: $v_B \approx 3.22$ (ft/s)