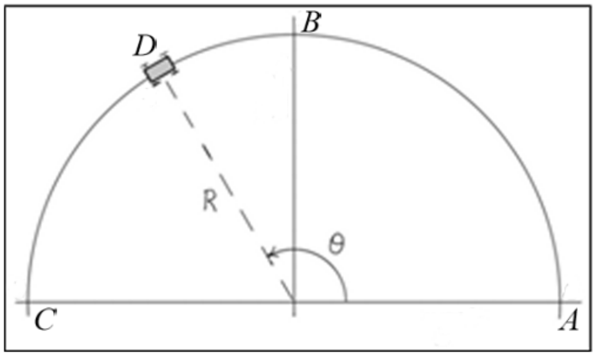
Elementary Dynamics
Exercises #3 – Curvilinear Motion: Nonstationary Unit Vectors

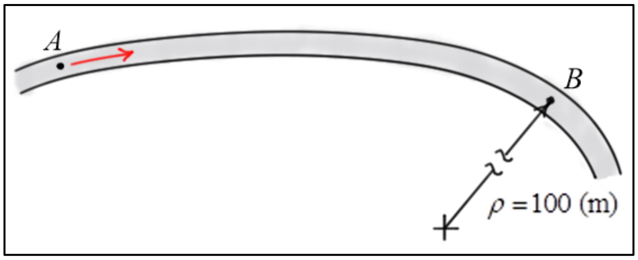
1. A car ***starts*** at *A* with ***velocity*** $v_A = 4\ \text{(m/s)}$ and ***increases*** its speed at a rate $\dot{v}(s) = 5 - 0.01s\ \text{(m/s}^2\text{)}$ as it travels from *A* to *D* along a circular track. The angle $\theta = 120\ \text{(deg)}$, and the radius of the path is $R = 100\ \text{(m)}$. Find a) a_t the ***tangential component*** of the acceleration of the car at *D*, b) $v(s)$ the velocity of the car as a function of s the distance traveled along the circular path, c) a_n the ***normal component*** of the acceleration of the car at *D*, and d) $\dot{\theta}$ and $\ddot{\theta}$ the first and second time derivatives of the angle θ at *D*.



Answers:

a) $a_t \approx 2.91\ \text{(m/s}^2\text{)}$; b) $v(s) = \sqrt{16 + 10\,s - \frac{s^2}{100}}\ \text{(m/s)}$; c) $a_n \approx 16.7\ \text{(m/s}^2\text{)}$;
d) $\dot{\theta} \approx 0.409\ \text{(r/s)}$; $\ddot{\theta} \approx 0.0291\ \text{(r/s}^2\text{)}$

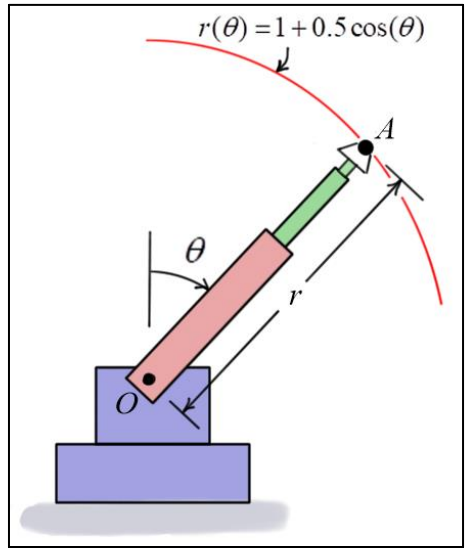
2. A car is travelling along a road from *A* to *B* as shown. The car has speed $v_A = 16\ \text{(m/s)}$ at *A* and changes speed at the rate $\dot{v}(s) = 3 - 0.02\,s\ \text{(m/s}^2\text{)}$ as it travels to *B*. Find: a) $v(s)$ the ***speed*** of the car as a ***function*** of distance s travelled along the road, and b) a_t and a_n the ***tangential*** and ***normal*** components of the ***acceleration*** of the car at *B*. The distance along the road from *A* to *B* is $120\ \text{(m)}$, and the ***radius of curvature*** of the path at *B* is $\rho = 100\ \text{(m)}$.



Answers:

a) $v(s) = \sqrt{256 - 6\,s - 0.02\,s^2}\ \text{(m/s)}$; b) $a_t = 0.6\ \text{(m/s}^2\text{)}$, $a_n = 6.88\ \text{(m/s}^2\text{)}$

3. As the telescopic robotic arm moves, end A moves so the distance from O to A is given by the ***function*** $r(\theta)=1+0.5\cos(\theta)$ (m). At the instant when $\theta=\pi/3$ (rad), the arm is rotating ***clockwise*** with $\dot{\theta}=0.6$ (rad/s) and $\ddot{\theta}=0.25$ (rad/s^2). Find: a) v_r and v_θ the ***radial*** and ***transverse components*** of the velocity of A at this instant, b) the ***angle*** ϕ between $\underset{\sim}{e}_\theta$ the ***transverse*** unit vector and $\underset{\sim}{e}_t$ the ***tangential*** unit vector, and c) a_r and a_θ the ***radial*** and ***transverse components*** of the acceleration of A at this instant.

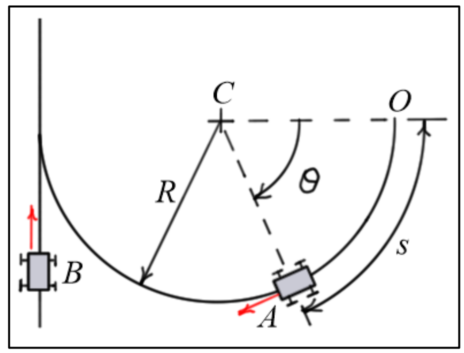


Answers:

a) $v_r=-0.260$ (m/s) , $v_\theta=0.75$ (m/s); b) $\phi=19.1$ (deg) $=0.333$ (rad)

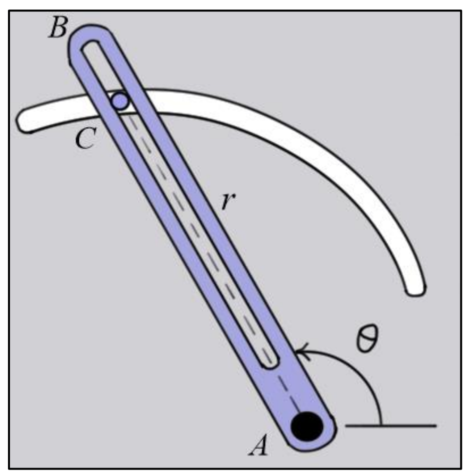
c) $a_r=-0.648$ (m/s^2) , $a_\theta=7.31\times10^{-4}$ (m/s^2)

4. Car A is approaching a highway on a circular entrance ramp of radius $R=200$ (ft). It starts from O at a speed of $v_O=30$ (ft/sec) and ***increases*** its speed at a rate $\dot{v}_A=0.02\,s$ (ft/sec^2). The distance s is measured in feet from point O. Find: a) $v(s)$ the speed of car A as a function of s the distance traveled along the circular path, and b) a_n and a_t the ***normal*** and ***tangential*** components of $\underset{\sim}{a}_A$ the acceleration of car A when $\theta=60$ (deg).



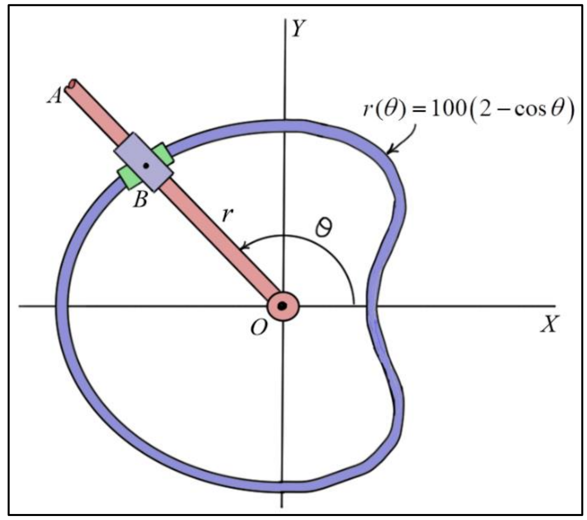
Answers: a) $v(s)=\sqrt{30^2+0.02\,s^2}$ (ft/sec); b) $a_n=8.89$ (ft/sec^2) and $a_t=4.19$ (ft/sec^2)

5. The slotted arm AB drives the pin C through the spiral groove described by the equation $r=1.5\theta$ (ft), where θ is in radians. The arm ***starts from rest*** at $\theta=60^{\circ}=\frac{\pi}{3}$ (rad) and is driven at an angular velocity of $\dot{\theta}=4t$ (rad/s), where time t is in seconds. Find: a) $\theta(t)$ the angle of arm AB as a function of time, b) v_r and v_θ the ***radial*** and ***transverse*** components of the ***velocity*** of C when $t=0.75$ (sec), and c) a_r and a_θ the ***radial*** and ***transverse*** components of the ***acceleration*** of C when $t=0.75$ (sec).



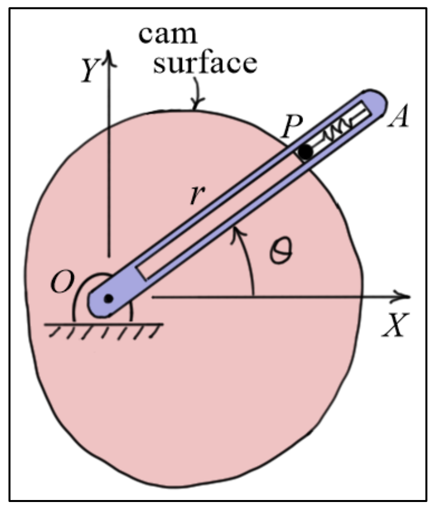
Answers: a) $\theta(t)=2t^2+\frac{\pi}{3}$ (rad); b) $v_r=4.5$ (ft/s); $v_\theta=9.77$ (ft/s); c) $a_r=-23.3$ (ft/s^2); $a_\theta=40.0$ (ft/s^2)

6. Bar OA rotates ***counterclockwise*** at a ***constant*** angular velocity of $\dot{\theta} = 5$ (r/s). The two pin-connected collars at B slide freely on OA and on the curved rod whose shape is described by the equation $r(\theta) = 100(2 - \cos\theta)$ (mm). When $\theta = 120$ degrees, find: a) v_r and v_θ the ***radial*** and ***transverse*** components of $\underset{\sim}{v}_B$ the ***velocity*** of B, b) ϕ the angle between the ***radial*** direction and the ***tangent*** to the curved path, and c) a_r and a_θ the ***radial*** and ***transverse*** components of $\underset{\sim}{a}_B$ the acceleration of B.



Answers:
a) $v_r = 433$ (mm/s); $v_\theta = 1250$ (mm/s); b) $\phi = 70.9$ (deg); c) $a_r = -7500$ (mm/s^2); $a_\theta = 4330$ (mm/s^2)

7. As the slotted arm OA rotates, the pin P slides along the surface of the cam. The surface of the cam is defined by the equation $r(\theta) = 2 + \sin(\theta)$ (ft). Given $\theta = 60$ (deg), $\dot{\theta} = 2$ (rad/s), and $\ddot{\theta} = 5$ (rad/s^2), find: a) v_r and v_θ the ***radial*** and ***transverse*** components of $\underset{\sim}{v}_P$ the velocity of pin P, and b) a_r and a_θ the ***radial*** and ***transverse*** components of $\underset{\sim}{a}_P$ the acceleration of pin P.



Answers: a) $v_r = 1$ (ft/s); $v_\theta \approx 5.73$ (ft/s);
b) $a_r \approx -12.4$ (ft/s^2); $a_\theta \approx 18.3$ (ft/s^2)