An Introduction to Three-Dimensional, Rigid Body Dynamics

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Volume II – Kinetics: Summary of Contents

Page Count	Examples	Suggested Exercises
278	46	50

Unit 1 – Inertia Matrices (Dyadics), Angular Momentum and Kinetic Energy

This unit defines ***moments*** and ***products of inertia*** for rigid bodies and shows how they are used to form ***inertia matrices*** (or ***dyadics***). Then it shows how to ***transform*** the ***components*** of ***inertia dyadics*** from ***one set*** of reference axes to ***another***. Finally, it defines ***angular momentum vectors*** and the ***kinetic energy function*** for rigid bodies and shows ***how*** to use inertia matrices to compute them.

Page Count	Examples	Suggested Exercises
29	5	6

Unit 2 – Newton/Euler Equations of Motion

This unit presents the ***Newton/Euler*** equations of motion for rigid bodies. These equations relate the ***kinematics*** of a body to the ***forces*** and ***torques*** acting upon it. The application of these equations focuses on ***individual bodies*** within a dynamic system using ***free body diagrams***. The analysis requires working knowledge of ***force systems***, ***kinematics*** of rigid bodies, and ***angular momentum***. The equations that result from the application of this method may be ***algebraic*** equations, ***differential*** equations, or a ***combination*** of both.

Page Count	Examples	Suggested Exercises
31	5	7

Unit 3 – Degrees of Freedom, Partial Velocities, and Generalized Forces

This unit defines the concepts of ***degrees of freedom***, ***generalized coordinates***, ***partial velocities***, ***partial angular velocities*** and ***generalized forces***. These concepts form an introduction to methods of treating systems with multiple bodies as ***systems*** rather than one body at a time as with the Newton/Euler equations of motion.

Page Count	Examples	Suggested Exercises
18	7	6

Unit 4 – Principle of Virtual Work and Lagrange's Equations

This unit discusses the ***Principle of Virtual Work*** for static systems and ***Lagrange's Equations*** for dynamic systems. The application of these methods relies heavily on the concepts of ***degrees of freedom***, ***generalized coordinates***, ***partial velocities***, ***partial angular velocities,*** and ***generalized forces*** discussed in Unit 3. As discussed in Unit 3, systems with multiple bodies will be analyzed as ***systems*** rather than one body at a time as with the Newton/Euler equations of motion.

Two addenda are provided to supplement the material presented in this unit. Addendum 1 shows the ***generalized forces*** associated with ***two equivalent force systems*** acting on a rigid body are ***equal***. Addendum 2 shows how to calculate the ***time derivative*** of a ***transformation matrix*** of a reference frame in terms of a ***skew-symmetric matrix*** related to the ***angular velocity*** of that frame.

Page Count	Examples	Suggested Exercises
29	8	7

Unit 5 – d'Alembert's Principle and Kane's Equations

This unit discusses the use of ***d'Alembert's principle*** and ***Kane's equations*** to develop the equations of motion of rigid body dynamic systems. The application of these methods relies heavily on the concepts of ***degrees of freedom***, ***generalized coordinates***, ***partial velocities***, ***partial angular velocities,*** and ***generalized forces*** discussed in Unit 3. As discussed in Units 3 and 4, systems with multiple bodies will be analyzed as ***systems*** rather than one body at a time as with the Newton/Euler equations of motion. Examples include an aircraft with two engines and an upright, two-wheeled bicycle.

Page Count	Examples	Suggested Exercises
66	7	8

Unit 6 – Lagrange's Equations, d'Alembert's Principle, and Kane's Equations for Systems with Constraints

This unit discusses the application of Lagrange's equations, d'Alembert's principle and Kane's equations to rigid body dynamic systems with ***constraints***. In Units 4 and 5 of this volume, the applications of Lagrange's equations and d'Alembert's principle were based on a set of ***independent generalized coordinates***, and the applications of Kane's equations were based on a set of ***independent generalized speeds***. There are systems for which it is ***inconvenient*** or ***impossible*** to eliminate surplus generalized coordinates or ***simply inconvenient*** to eliminate generalized speeds from the analysis. For these systems, the application of each of these three methods can be supplemented with the use of ***Lagrange multipliers***.

The resulting equations of motion are a set of ***differential*** and ***algebraic*** equations. The Lagrange multipliers are ***algebraic unknowns*** related to the forces and torques required to maintain the constraints. An Addendum to this unit explores this connection.

In the analysis that follows, constraint equations are assumed to be ***equality constraints***, that is, some function of the generalized coordinates and/or generalized speeds is equal to zero. The case of inequality constraints is not considered.

Page Count	Examples	Suggested Exercises
30	4	7

Unit 7 – Introduction to Modeling Mechanical System Kinetics using MATLAB® Scripts, Simulink®, and SimMechanics®

This unit provides and introduction to ***modeling*** mechanical system kinetics using ***MATLAB scripts***, Simulink ***models***, and ***SimMechanics models***. For an introduction to these modeling techniques, see Unit 10 of Volume I for applications in mechanical systems kinematics. The examples presented in this unit assume the reader is familiar with the programming concepts presented in Volume I.

As presented in Volume I, MATLAB scripts are ***text-based programs*** written in the MATLAB programming language. Simulink models are ***block-diagram-based programs*** that run in the MATLAB environment. SimMechanics models are ***block-diagram-based, multibody dynamics programs*** that run in the MATLAB/Simulink environment. MATLAB scripts can be used ***alone*** or ***in conjunction with*** Simulink and SimMechanics models.

Three of the six models presented herein simulate the ***free motion*** of an ***upright bicycle***. The models are developed using the equations of motion developed in Unit 5 of this volume.

Page Count	Examples	Models	Suggested Exercises
48	3	6	4

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Unit 8 – Basic Concepts of Linearization, Stability, Mode Shapes, and Natural Frequencies

The equations of motion of rigid body systems can be ***linear*** or ***nonlinear***. If the equations are ***nonlinear***, it may be possible to ***linearize*** them about some ***steady-state*** (equilibrium) positions. This unit describes how to find equilibrium positions (if they exist) and how to linearize the equations of motion about those positions. It also shows how to use the linear, approximate equations of motion to determine the ***stability*** of small motions about the equilibrium position. Finally, for stable equilibrium positions, this unit shows how to calculate the ***undamped natural frequencies*** and ***mode shapes*** associated with those positions. Examples are given for one and two degree-of-freedom systems. Extension to multi-degree-of-freedom systems is straightforward. MATLAB® is used to calculate eigenvalues and eigenvectors associated with the analysis.

Page Count	Examples	Suggested Exercises
27	7	5

References:

1. L. Meirovitch, *Methods of Analytical Dynamics*, McGraw-Hill, 1970.
2. T.R. Kane, P.W. Likins, and D.A. Levinson, *Spacecraft Dynamics*, McGraw-Hill, 1983
3. T.R. Kane and D.A. Levinson, *Dynamics: Theory and Application*, McGraw-Hill, 1985
4. R.L. Huston, *Multibody Dynamics*, Butterworth-Heinemann, 1990
5. H. Baruh, *Analytical Dynamics*, McGraw-Hill, 1999
6. H. Josephs and R.L. Huston, *Dynamics of Mechanical Systems*, CRC Press, 2002
7. J.H. Williams, Jr., *Fundamentals of Applied Dynamics*, John Wiley & Sons, Inc., 1996
8. R.C. Hibbeler, *Engineering Mechanics: Dynamics*, 13^{th} Ed., Pearson Prentice Hall, 2013
9. J.L. Meriam and L.G. Craig, *Engineering Mechanics: Dynamics*, 3^{rd} Ed, 1992
10. F.P. Beer and E.R. Johnston, Jr. *Vector Mechanics for Engineers: Dynamics*, 4^{th} Ed, 1984
11. M.L. Boas, Mathematical Methods in the Physical Sciences, John Wiley & Sons, Inc., 1966
12. R. Bronson, *Matrix Methods – An Introduction*, Academic Press, 1970
13. Bicycle references:
    a. B.W. Kostich, "Development of Empirical and Virtual Tools for the Study of Bicycle Safety", MS Thesis, Western Michigan University, 2017
    b. J.K. Moore, "Human Control of a Bicycle", Ph.D. Dissertation, University of California-Davis, 2012
    c. J.P. Meijaard, J.M. Papadopoulos, A. Ruina, and A.L. Schwab, "Linearized Dynamics Equations for the Balance and Steer of a Bicycle: a Benchmark and Review", *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 463(2084), 1955–1982, 2007
    d. P. Basu-Mandal, A. Chatterjee, and J.M. Papadopoulos, "Hands-free Circular Motions of a Benchmark Bicycle. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 463(March), 1983–2003, 2007
14. J.W. Kamman, "Effects of Quadratic Nonlinearities on the Response of Individual Modes of Towed Cable Systems Under Tow Point Excitation," CSS TM 628-92, Naval Surface Warfare Center, Nov 1992, (DTIC AD-A257619)