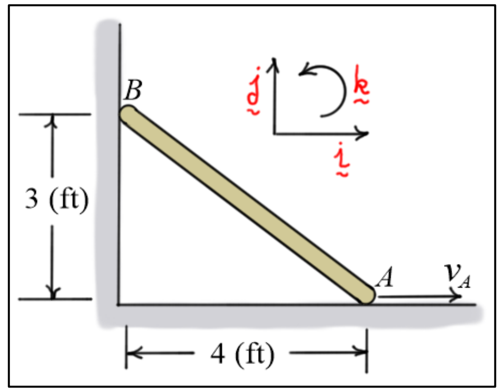
Elementary Dynamics
Exercises #7 – Two-Dimensional Rigid Body Kinematics

1. Bar *AB* rests against a vertical wall as shown. At the instant shown, the velocity of *A* is ***constant*** $\underset{\sim}{v}_A = 9\underset{\sim}{i}$ (ft/s). At this instant, find: a) $\underset{\sim}{v}_B$ the velocity of *B* and $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB*, and b) $\underset{\sim}{a}_B$ the acceleration of *B* and $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB*.

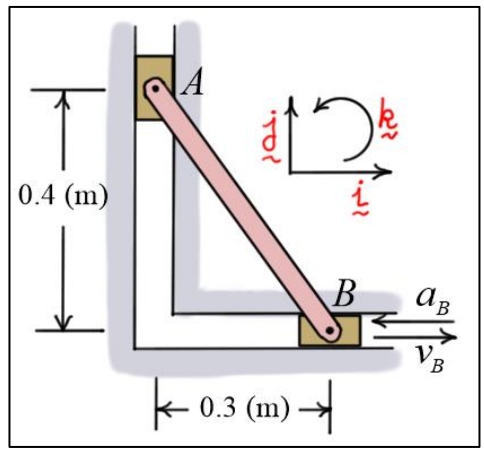


Answers:

a) $\underset{\sim}{v}_B \approx -12\underset{\sim}{j}$ (ft/s) and $\underset{\sim}{\omega}_{AB} \approx 3\underset{\sim}{k}$ (rad/s)

b) $\underset{\sim}{a}_B \approx -75\underset{\sim}{j}$ (ft/s^2) and $\underset{\sim}{\alpha}_{AB} \approx 12\underset{\sim}{k}$ (rad/s^2)

2. Bar *AB* has its ends constrained to move in the horizontal and vertical slots. At the instant shown, the point *B* has velocity $\underset{\sim}{v}_B = 2\underset{\sim}{i}$ (m/sec) and acceleration $\underset{\sim}{a}_B = -5\underset{\sim}{i}$ (m/sec^2). At this instant, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\underset{\sim}{v}_A$ the velocity of *A*, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\underset{\sim}{a}_A$ the acceleration of *A*.

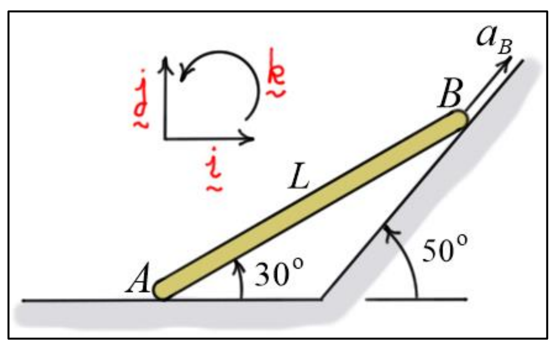


Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 5\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_A \approx -1.5\underset{\sim}{j}$ (m/s);

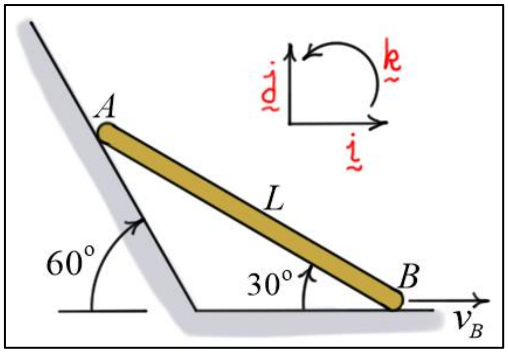
b) $\underset{\sim}{\alpha}_{AB} \approx 6.25\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_A \approx -11.9\underset{\sim}{j}$ (m/s^2)

3. The ends of bar *AB* slide along the horizontal and inclined surfaces. At the instant shown, the angular velocity of *AB* is $\underset{\sim}{\omega}_{AB} = 2\underset{\sim}{k}$ (rad/s), and the acceleration of end *B* is $a_B = 1$ (ft/s^2) up the inclined plane. At this instant, find $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\underset{\sim}{a}_A$ the acceleration of end *A*. The length of *AB* is $L = 5$ (ft).



Answers: $\underset{\sim}{\alpha}_{AB} \approx 2.49\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_A \approx 24.2\underset{\sim}{i}$ (ft/s^2)

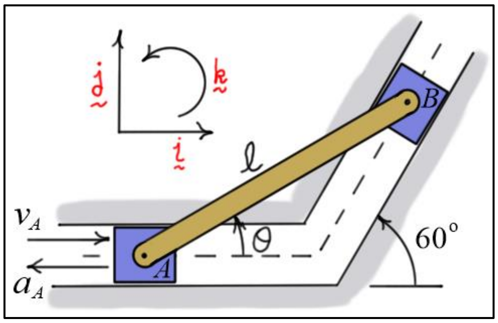
4. The ends of bar *AB* of length $L = 10$ (in) slide along the horizontal and inclined surfaces as shown. At the instant shown, end *B* has velocity $\underset{\sim}{v}_B = 20\underset{\sim}{i}$ (in/sec). a) Using the ***relative velocity equation***, find $\underset{\sim}{v}_A$ the velocity of *A* and $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* at this instant. b) Repeat part (a) using the concept of ***instantaneous centers*** of zero velocity.



Answers:

$\left|\underset{\sim}{v}_A\right| \approx 20$ (in/s); $\underset{\sim}{v}_A \approx 10\underset{\sim}{i} - 17.3\underset{\sim}{j}$ (in/s); $\underset{\sim}{\omega}_{AB} \approx 2\underset{\sim}{k}$ (rad/s)

5. The figure shows bar *AB* of length $\ell = 2$ (ft) whose ends slide along the horizontal and inclined surfaces. At the instant shown, angle $\theta = 30$ (deg), the velocity of *A* is $\underset{\sim}{v}_A = 5\underset{\sim}{i}$ (ft/s), and the acceleration of *A* is $\underset{\sim}{a}_A = -10\underset{\sim}{i}$ (ft/s^2). At this instant, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\underset{\sim}{v}_B$ the velocity of *B*, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\underset{\sim}{a}_B$ the acceleration of *B*.

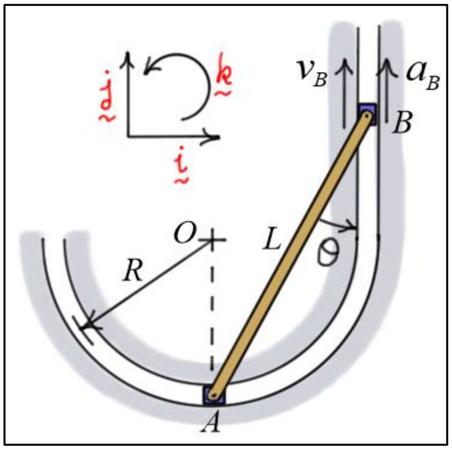


Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 2.5\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_B \approx 5\left(\cos(60)\underset{\sim}{i} + \sin(60)\underset{\sim}{j}\right)$ (ft/s)

b) $\underset{\sim}{\alpha}_{AB} \approx -8.61\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_B \approx -24.4\left(\cos(60)\underset{\sim}{i} + \sin(60)\underset{\sim}{j}\right)$ (ft/s^2)

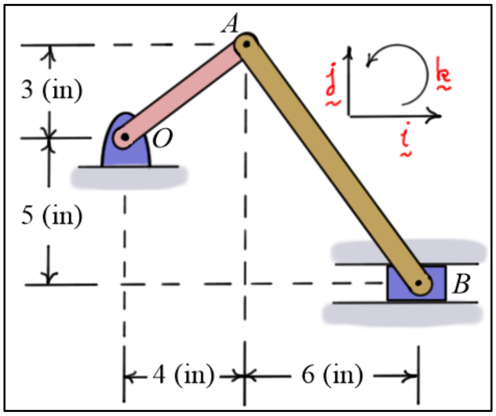
6. At the instant shown, end *A* of bar *AB* moves along a circular slot while end *B* moves along the straight slot. The radius of the circular slot is $R = 0.2$ (m), the length of the bar is $L = 0.4$ (m), and the angle $\theta = 30$ (deg). At the instant shown, the velocity of *B* is $\underset{\sim}{v}_B = 5\underset{\sim}{j}$ (m/s) and the acceleration of *B* is $\underset{\sim}{a}_B = 10\underset{\sim}{j}$ (m/s^2). a) Using the concept of ***instantaneous centers*** of zero velocity, find $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\underset{\sim}{v}_A$ the velocity of *A*. b) Using the ***relative acceleration equation,*** find $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\underset{\sim}{a}_A$ the acceleration of *A*.



Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 25\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_A \approx 8.66\underset{\sim}{i}$ (m/s); b) $\underset{\sim}{\alpha}_{AB} \approx -742\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_A \approx -132\underset{\sim}{i} + 375\underset{\sim}{j}$ (m/s^2)

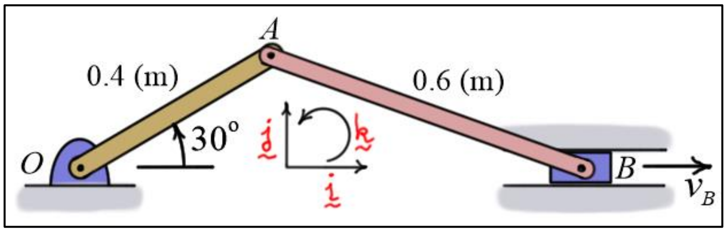
7. The figure shows a slider-crank mechanism *OAB*. Crank *OA* is driven at a ***constant*** angular velocity of $\underset{\sim}{\omega}_{OA} = -10\underset{\sim}{k}$ (rad/sec). a) Using the ***relative velocity equation,*** find $\underset{\sim}{\omega}_{AB}$ the angular velocity of the connecting bar *AB* and $\underset{\sim}{v}_B$ the velocity of the slider *B*. b) Repeat part (a) using the concept of ***instantaneous centers*** of zero velocity.



Answers:

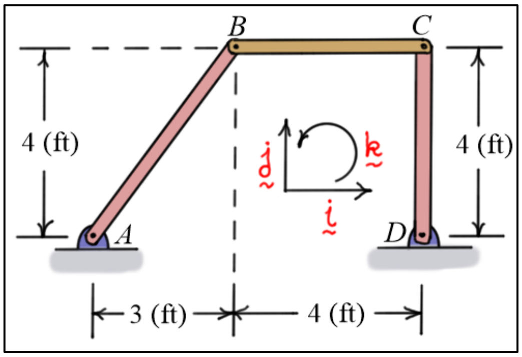
$\underset{\sim}{\omega}_{AB} \approx 6.67\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_B \approx 83.3\underset{\sim}{i}$ (in/s)

8. The figure shows slider-crank mechanism *OAB*. At the instant shown, the velocity of *B* is $\underset{\sim}{v}_B = 3\underset{\sim}{i}$ (m/s). Using the concept of ***instantaneous centers*** of zero velocity, find $\underset{\sim}{\omega}_{AB}$ the angular velocity of the connecting rod *AB* and $\underset{\sim}{v}_A$ the velocity of *A* at this instant.



Answers: $\underset{\sim}{\omega}_{AB} \approx 5.7\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_A \approx 3.72\left(\sin(30)\underset{\sim}{i} - \cos(30)\underset{\sim}{j}\right)$

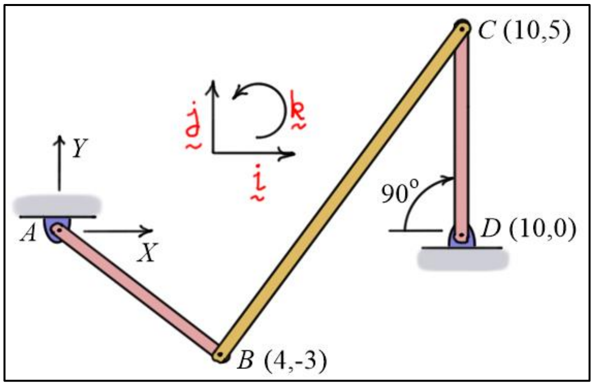
9. At the instant shown, the ***angular velocity*** of bar *AB* of the four-bar mechanism *ABCD* is $\underset{\sim}{\omega}_{AB} = 10\underset{\sim}{k}$ (rad/s). Using the concept of ***instantaneous centers*** of zero velocity, find $\underset{\sim}{\omega}_{BC}$ and $\underset{\sim}{\omega}_{CD}$ the angular velocities of bars *BC* and *CD*.



Answers:

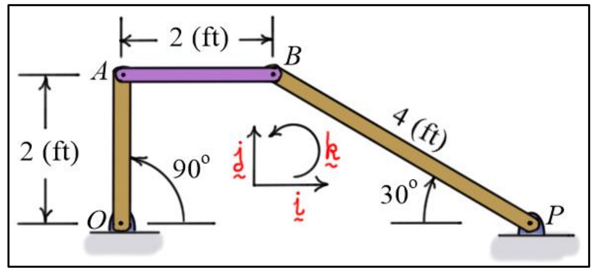
$\underset{\sim}{\omega}_{BC} \approx -7.5\underset{\sim}{k}$ (rad/s); $\underset{\sim}{\omega}_{CD} \approx 10\underset{\sim}{k}$ (rad/s)

10. The figure shows a four-bar mechanism *ABCD*. Point *A* is located at the origin, and points *B*, *C* and *D* have the coordinates shown. At the instant shown, link *AB* has a ***constant*** angular velocity of $\underset{\sim}{\omega}_{AB} = 9\underset{\sim}{k}$ (rad/s). a) Using the ***relative velocity equation***, find $\underset{\sim}{\omega}_{BC}$ and $\underset{\sim}{\omega}_{CD}$ the angular velocities of links *BC* and *CD*. b) Repeat part (a) using the concept of ***instantaneous centers*** of zero velocity.



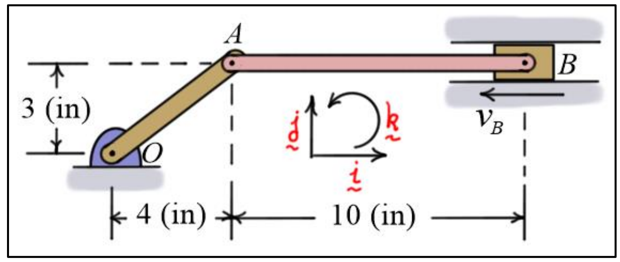
Answers: $\underset{\sim}{\omega}_{BC} \approx -6\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{\omega}_{CD} \approx -15\underset{\sim}{k}$ (rad/s)

11. The figure shows a four-bar mechanism *OABP*. At the instant shown, the angular velocity of the crank *OA* is $\underset{\sim}{\omega}_{OA} = 5\underset{\sim}{k}$ (rad/s). Using the concept of ***instantaneous centers*** of zero velocity, find $\underset{\sim}{\omega}_{AB}$ the angular velocity of connecting bar *AB*.



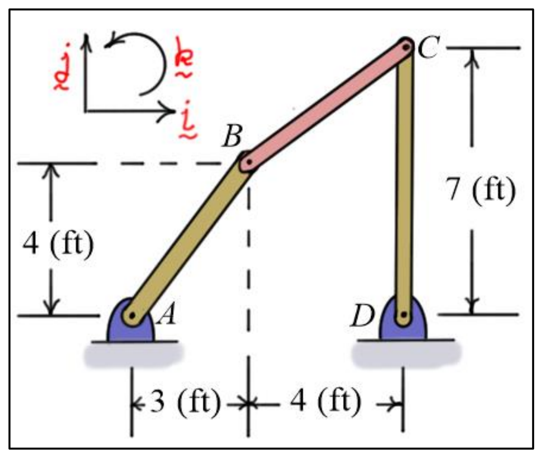
Answer: $\underset{\sim}{\omega}_{AB} \approx -8.66\underset{\sim}{k}$ (rad/s)

12. The figure shows a slider-crank mechanism *OAB*. At the instant shown, the velocity of slider *B* is $\underset{\sim}{v}_B = -6\underset{\sim}{i}$ (in/s). a) Using the ***relative velocity equation***, find $\underset{\sim}{\omega}_{AB}$ the angular velocity of connecting rod *AB* and $\underset{\sim}{\omega}_{OA}$ the angular velocity of crank *OA*. b) Repeat part (a) using the concept of ***instantaneous centers*** of zero velocity.



Answers: $\underset{\sim}{\omega}_{AB} \approx -0.8\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{\omega}_{OA} \approx 2\underset{\sim}{k}$ (rad/s)

13. At the instant shown, the angular velocity of bar *AB* of the four-bar mechanism *ABCD* is $\underset{\sim}{\omega}_{AB} = 10\underset{\sim}{k}$ (rad/s). Using the ***relative velocity equation***, find a) $\underset{\sim}{v}_B$ the velocity of point *B*, and b) $\underset{\sim}{\omega}_{BC}$ and $\underset{\sim}{\omega}_{CD}$ the angular velocities of bars *BC* and *CD*.

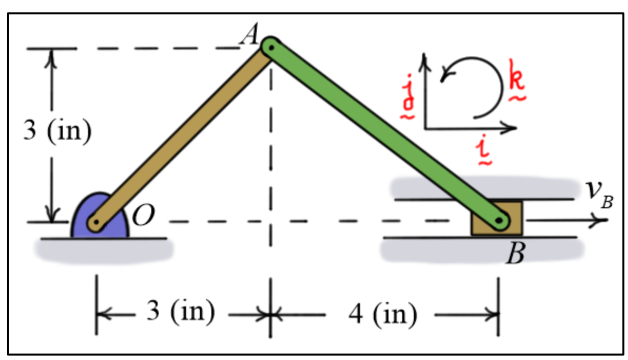


Answers:

a) $\underset{\sim}{v}_B \approx -40\underset{\sim}{i} + 30\underset{\sim}{j}$ (ft/s)

b) $\underset{\sim}{\omega}_{BC} \approx -7.5\underset{\sim}{k}$ (rad/s); $\underset{\sim}{\omega}_{CD} \approx 2.5\underset{\sim}{k}$ (rad/s)

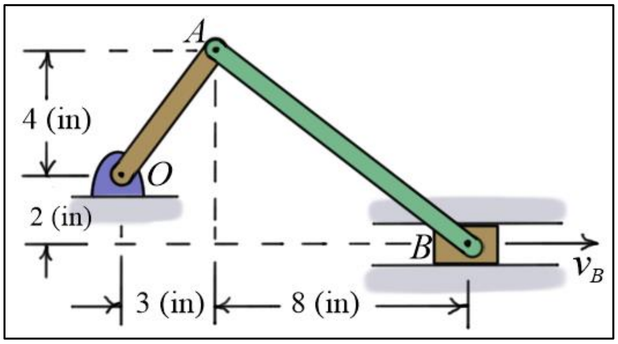
14. The figure shows a slider-crank mechanism *OAB*. The slider *B* moves at a ***constant velocity*** $\underset{\sim}{v}_B = 10.5\underset{\sim}{i}$ (in/s). At the instant shown, find: a) $\underset{\sim}{\omega}_{OA}$ and $\underset{\sim}{\omega}_{AB}$ the angular velocities of the two bars, and b) $\underset{\sim}{\alpha}_{OA}$ and $\underset{\sim}{\alpha}_{AB}$ the angular accelerations of the bars.



Answers:

a) $\underset{\sim}{\omega}_{OA} = -2\underset{\sim}{k}$ (rad/s), $\underset{\sim}{\omega}_{AB} = +1.5\underset{\sim}{k}$ (rad/s); b) $\underset{\sim}{\alpha}_{OA} \approx -3.25\underset{\sim}{k}$ (rad/s^2), $\underset{\sim}{\alpha}_{AB} \approx 3.75\underset{\sim}{k}$ (rad/s^2)

15. The figure shows a slider-crank mechanism *OAB*. The slider has ***constant*** velocity $\underset{\sim}{v}_B = 20\,\underset{\sim}{i}$ (in/sec). At the instant shown, find: a) $\underset{\sim}{\omega}_{OA}$ and $\underset{\sim}{\omega}_{AB}$ the angular velocities of *OA* and *AB*, and b) $\underset{\sim}{\alpha}_{OA}$ and $\underset{\sim}{\alpha}_{AB}$ the angular accelerations of *OA* and *AB*.

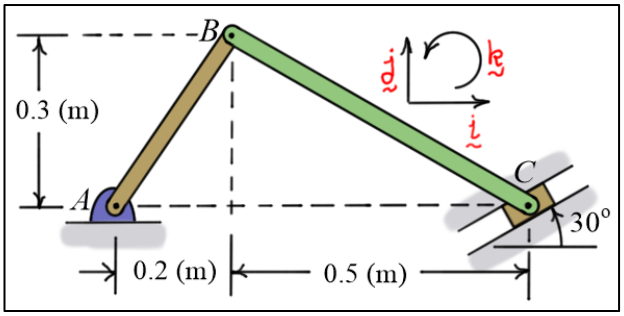


Answers:

a) $\underset{\sim}{\omega}_{OA} = -3.2\,\underset{\sim}{k}$ (rad/sec) and $\underset{\sim}{\omega}_{AB} = 1.2\,\underset{\sim}{k}$ (rad/sec);

b) $\underset{\sim}{\alpha}_{OA} \approx -2.88\,\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{\alpha}_{AB} \approx 5.12\,\underset{\sim}{k}$ (rad/s^2)

16. The figure shows a slider-crank mechanism *ABC*. Bar *AB* has a ***constant*** angular velocity of $\underset{\sim}{\omega}_{AB} = -10\,\underset{\sim}{k}$ (rad/sec). Using the equations for relative velocity and acceleration, find: a) $\underset{\sim}{\omega}_{BC}$ the angular velocity of *BC* and $\underset{\sim}{v}_C$ the velocity of *C*, and b) $\underset{\sim}{\alpha}_{BC}$ the angular acceleration of *BC* and $\underset{\sim}{a}_C$ the acceleration of *C*.

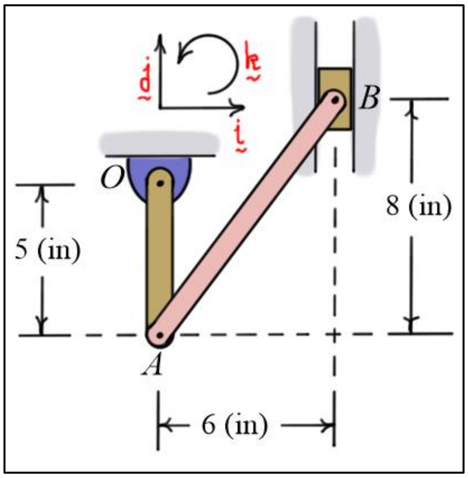


Answers:

a) $\underset{\sim}{\omega}_{BC} \approx 11.4\,\underset{\sim}{k}$ (rad/s) and $\underset{\sim}{v}_C \approx 7.42\left(\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right)$ (m/s)

b) $\underset{\sim}{\alpha}_{BC} \approx -178\,\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_C \approx -160\left(\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right)$ (m/s^2)

17. The figure shows a slider-crank mechanism *OAB*. Crank *OA* is driven at a ***constant*** rate of $\underset{\sim}{\omega}_{OA} = 8\,\underset{\sim}{k}$ (rad/sec). At the instant shown, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\underset{\sim}{v}_B$ the velocity of *B*, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\underset{\sim}{a}_B$ the acceleration of *B*.

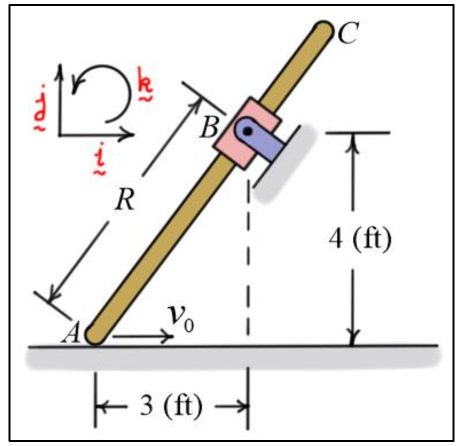


Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 5\,\underset{\sim}{k}$ (rad/sec) and $\underset{\sim}{v}_B \approx 30\,\underset{\sim}{j}$ (in/s)

b) $\underset{\sim}{\alpha}_{AB} \approx -18.8\,\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{a}_B \approx 7.5\,\underset{\sim}{j}$ (in/s^2)

18. Bar AC rests on the horizontal plane at A and slides through a ***smooth collar*** at B. The collar rotates freely to allow the bar to rotate as A moves at a ***constant*** speed of $\underset{\sim}{v}_0 = 10\,\underset{\sim}{i}$ (ft/s). The variable distance from A to B is R. At the instant shown, find: a) $\underset{\sim}{\omega}_{AC}$ the angular velocity of AC and $\dot{R}$ the time rate of change of the distance R, and b) $\underset{\sim}{\alpha}_{AC}$ the angular acceleration of AC and $\ddot{R}$ the time rate of change of $\dot{R}$.

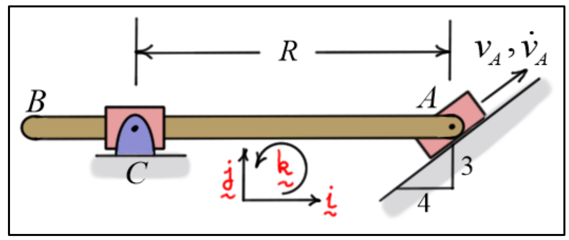


Answers:

a) $\underset{\sim}{\omega}_{AC} \approx 1.6\underset{\sim}{k}$ (rad/s) ; $\dot{R} \approx -6.0$ (ft/s)

b) $\underset{\sim}{\alpha}_{AC} \approx 3.84\underset{\sim}{k}$ (rad/s^2) ; $\ddot{R} \approx 12.8$ (ft/s^2)

19. Bar AB slides through the collar at C while its end A moves up the inclined plane. The collar rotates freely to allow the bar to rotate as A moves up the plane. At the instant shown, the velocity and acceleration of A are $v_A = 10$ (ft/s) and $\dot{v}_A = 5$ (ft/s^2), and the variable distance from C to A is $R = 2$ (ft).

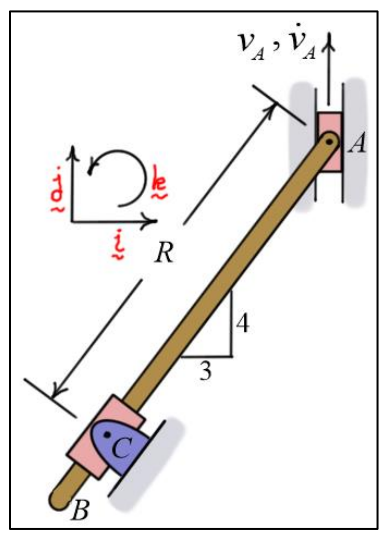


At this instant, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of AB and $\dot{R}$ the time rate of change of R, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB and $\ddot{R}$ the time rate of change of $\dot{R}$.

Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 3\underset{\sim}{k}$ (rad/s) and $\dot{R} \approx 8$ (ft/s) ; b) $\underset{\sim}{\alpha}_{AB} \approx -22.5\underset{\sim}{k}$ (rad/s^2) and $\ddot{R} \approx 22$ (ft/s^2)

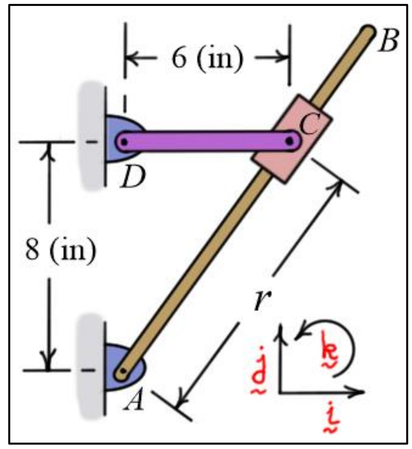
20. Bar AB slides through a collar at C while its end A moves in the vertical slot. The collar rotates freely to allow the bar to rotate as A moves up. At the instant shown, the velocity and acceleration of A are $\underset{\sim}{v}_A = 12.5\,\underset{\sim}{j}$ (ft/s) and $\dot{\underset{\sim}{v}}_A = 5\,\underset{\sim}{j}$ (ft/s^2), and the variable distance from C to A is $R = 5$ (ft). At this instant, find a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of AB and $\dot{R}$ the time rate of change of R, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB and $\ddot{R}$ the time rate of change of $\dot{R}$.



Answers: a) $\underset{\sim}{\omega}_{AB} = 1.5\,\underset{\sim}{k}$ (rad/s), $\dot{R} = 10$ (ft/s) ;

b) $\underset{\sim}{\alpha}_{AB} \approx -5.4\underset{\sim}{k}$ (rad/s^2) and $\ddot{R} \approx 15.3$ (ft/s^2)

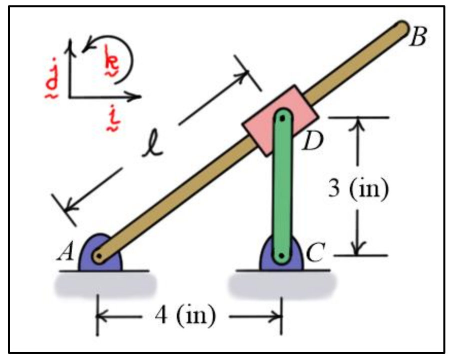
21. The system shown consists of two bars *AB* and *CD* connected by a collar at *C*. Bar *AB* is free to slide through the collar as it rotates at a ***constant*** rate of $\underset{\sim}{\omega}_{AB} = 18\,\underset{\sim}{k}\ \text{(rad/s)}$, and bar *CD* is pinned to the collar at *C*. The variable distance from *A* to *C* is represented by the symbol r. At the instant shown, find: a) $\underset{\sim}{\omega}_{CD}$ the angular velocity of *CD* and $\dot{r}$ the time rate of change of the distance r, and b) $\underset{\sim}{\alpha}_{CD}$ the angular acceleration of *CD* and $\ddot{r}$ the time rate of change of $\dot{r}$.



Answers:

a) $\underset{\sim}{\omega}_{CD} \approx 50\,\underset{\sim}{k}\ \text{(rad/s)}$ and $\dot{r} \approx 20\ \text{(ft/s)}$; b) $\underset{\sim}{\alpha}_{CD} \approx -933\underset{\sim}{k}\ \text{(rad/s}^2\text{)}$ and $\ddot{r} \approx -853\ \text{(ft/s}^2\text{)}$

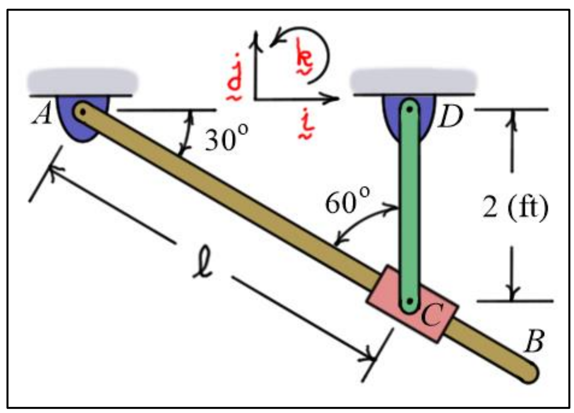
22. The system shown consists of two bars *AB* and *CD* and a collar at *D*. The collar is pinned to bar *CD* and is free to slide along and rotate with bar *AB*. The variable distance between *A* and *D* is ℓ. Bar *CD* rotates at a ***constant rate*** $\underset{\sim}{\omega}_{CD} = 10\,\underset{\sim}{k}\ \text{(rad/s)}$. At the instant shown, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\dot{\ell}$ the time rate of change of ℓ, and b) $\underset{\sim}{\alpha}_{AB}$ the ***angular acceleration*** of *AB*, and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.



Answers:

a) $\underset{\sim}{\omega}_{AB} = 3.6\,\underset{\sim}{k}\ \text{(r/s)}$ and $\dot{\ell} = -24\ \text{(in/s)}$; b) $\underset{\sim}{\alpha}_{AB} \approx -13.4\underset{\sim}{k}\ \text{(rad/s}^2\text{)}$ and $\ddot{\ell} \approx -115\ \text{(in/s}^2\text{)}$

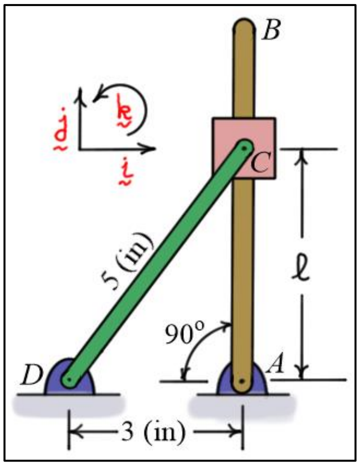
23. The system shown consists of two bars *AB* and *CD* and a collar at *C*. The collar is pinned to bar *CD* and is free to slide along and rotate with bar *AB*. The ***variable length*** between *A* and *C* is ℓ. Bar *CD* rotates at a ***constant rate*** of $\underset{\sim}{\omega}_{CD} = 5\underset{\sim}{k}\ \text{(rad/s)}$. At the instant shown when $\ell = 4\ \text{(ft)}$, find: a) $\underset{\sim}{\omega}_{AB}$ the angular velocity of *AB* and $\dot{\ell}$ the time derivative of ℓ, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of *AB* and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.



Answers:

a) $\underset{\sim}{\omega}_{AB} \approx 1.25\underset{\sim}{k}\ \text{(rad/s)}$ and $\dot{\ell} \approx 8.66\ \text{(ft/s)}$; b) $\underset{\sim}{\alpha}_{AB} \approx 5.41\underset{\sim}{k}\ \text{(rad/s}^2\text{)}$ and $\ddot{\ell} \approx -18.8\ \text{(ft/s}^2\text{)}$

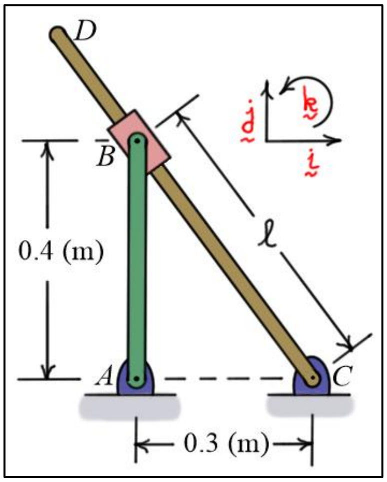
24. The system shown consists of two bars *AB* and *CD* connected by a collar at *C*. Bar *AB* is free to slide through the collar as it rotates, and bar *CD* is pinned to the collar. Length ℓ represents the ***variable distance*** from *A* to *C*. At the instant shown, length $\ell = 4$ (in) and the angular velocity of *AB* is ***constant*** $\underset{\sim}{\omega}_{AB} = 5\,\underset{\sim}{k}$ (rad/s). At this instant, find: a) $\underset{\sim}{\omega}_{CD}$ the angular velocity of *CD* and $\dot{\ell}$ the first derivative of the distance ℓ, and b) $\underset{\sim}{\alpha}_{CD}$ the angular acceleration of *CD* and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.



Answers:

a) $\underset{\sim}{\omega}_{CD} \approx 5\underset{\sim}{k}$ (rad/s) and $\dot{\ell} \approx 15$ (in/s); b) $\underset{\sim}{\alpha}_{CD} \approx 18.8\underset{\sim}{k}$ (rad/s^2) and $\ddot{\ell} \approx 56.3$ (in/s^2)

25. The system shown consists of two bars *AB* and *CD* connected by a collar at *B*. Bar *CD* is free to slide through the collar as it rotates, and bar *AB* is pinned to the collar. The length ℓ represents the ***variable distance*** from *C* to *B*. At the instant shown, length $\ell = 0.5$ (m) and the angular velocity of *AB* is ***constant*** $\underset{\sim}{\omega}_{AB} = 10\,\underset{\sim}{k}$ (rad/s). At this instant, find: a) $\underset{\sim}{\omega}_{CD}$ the angular velocity of *CD* and $\dot{\ell}$ the time derivative of the length ℓ, and b) $\underset{\sim}{\alpha}_{CD}$ the angular acceleration of *CD* and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.



Answers:

a) $\underset{\sim}{\omega}_{CD} \approx 6.4\underset{\sim}{k}$ (rad/s) and $\dot{\ell} \approx 2.4$ (m/s); $\underset{\sim}{\alpha}_{CD} \approx -13.4\underset{\sim}{k}$ (rad/s^2) and $\ddot{\ell} \approx -11.5$ (m/s^2)