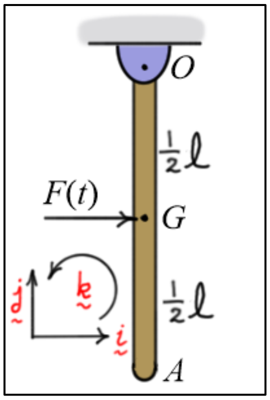
Elementary Dynamics
Exercises #10 – Two-Dimensional, Rigid Body Kinetics: Impulse, Momentum and Energy

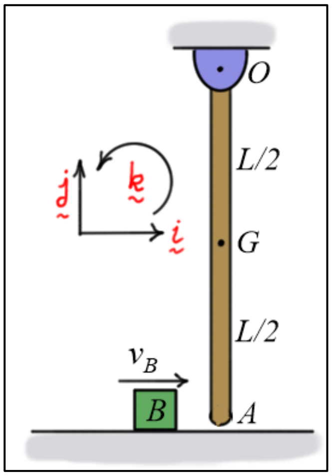
1. The long slender bar *OA* hangs down from the fixed support at *O*. The bar has mass $m = 8\ \text{(kg)}$ and length $\ell = 2\ \text{(m)}$. *OA* is initially at rest in the vertical position when the impulsive force $F(t)$ is applied. $F(t)$ acts for $0.03\ \text{(sec)}$ and has an average value of $1000\ \text{(N)}$. Find $\underset{\sim}{\omega}_{OA}$ the angular velocity of *OA* just after the $0.03\ \text{(sec)}$ interval.



   Answer:

   $\underset{\sim}{\omega}_{OA} \approx 2.81\underset{\sim}{k}\ \text{(rad/s)}$

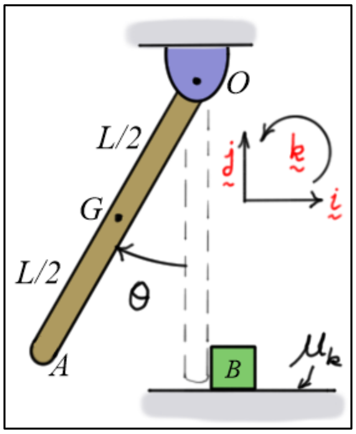
2. The system shown consists of a slender bar *OA* and a small block *B*. Bar *OA* weighs $W_{OA} = 10\ \text{(lb)}$ and has length $L = 3\ \text{(ft)}$. Block *B* weighs $W_B = 2\ \text{(lb)}$. The bar is initially at rest when the block strikes it moving at a speed of $v_B = 30\ \text{(ft/s)}$. Given the coefficient of restitution is $e = 0.4$ for the collision find: a) $\left(\underset{\sim}{v}_B\right)_a$ the velocity of *B*, and b) $\left(\underset{\sim}{\omega}_{OA}\right)_a$ the angular velocity of the bar just after impact.



   Answer:

   a) $\left(\underset{\sim}{v}_B\right)_a \approx 3.75\underset{\sim}{i}\ \text{(ft/s)}$; b) $\left(\underset{\sim}{\omega}_{OA}\right)_a \approx 5.25\underset{\sim}{k}\ \text{(rad/s)}$

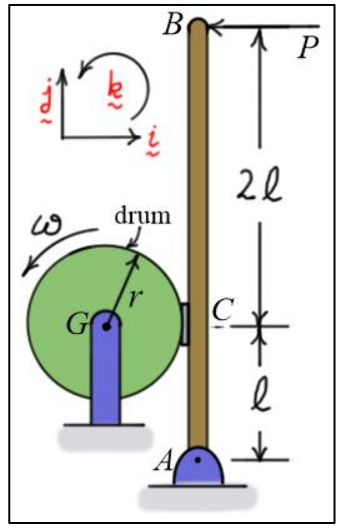
3. The system shown consists of a slender bar *OA* and a small block *B*. The bar has weight $W_{OA} = 10\ \text{(lb)}$ and length $L = 3\ \text{(ft)}$. Block *B* has weight $W_B = 2\ \text{(lb)}$. The bar is released from rest at $\theta = 30^o$ and rotates down to strike the block when it is vertical. The block is initially at rest. a) Find $\underset{\sim}{\omega}_{OA}$ the angular velocity of bar *OA* just before it strikes *B*. b) Find $\left(\underset{\sim}{\omega}_{OA}\right)_a$ the angular velocity of *OA* and $\left(\underset{\sim}{v}_B\right)_a$ the velocity of *B* just after the impact given the coefficient of restitution is $e = 0.4$ for the collision. c) Find how long it takes for the block to stop given the kinetic coefficient of friction between the block and the plane is $\mu_k = 0.3$.



   Answers:

   a) $\underset{\sim}{\omega}_{OA} \approx 2.08\underset{\sim}{k}\ \text{(rad/s)}$; b) $\left(\underset{\sim}{\omega}_{OA}\right)_a \approx 0.987\underset{\sim}{k}\ \text{(rad/s)}$ and $\left(\underset{\sim}{v}_B\right)_a \approx 5.45\underset{\sim}{i}\ \text{(ft/s)}$; c) $\Delta t \approx 0.564\ \text{(sec)}$

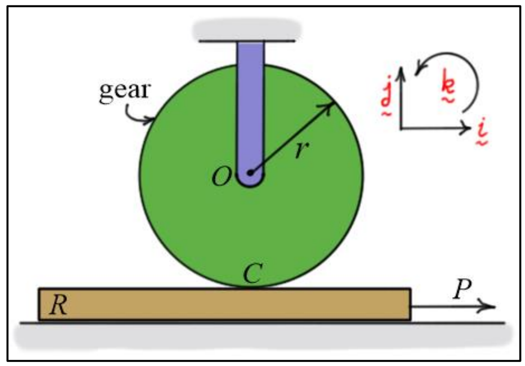
4. The drum has mass $m = 50\ \text{(kg)}$, a radius of $r = 0.25\ \text{(m)}$, and a radius of gyration about its mass center *G* of $k_G = 0.3\ \text{(m)}$. When the brake *AB* is applied, the drum has an angular velocity of $\underset{\sim}{\omega} = 40\underset{\sim}{k}\ \text{(r/s)}$. The breaking force $P = 100\ \text{(N)}$, the length $\ell = 0.5\ \text{(m)}$, and the coefficient of kinetic friction between the brake and the drum is $\mu_k = 0.5$. Neglect the weight and thickness of the handle. a) Using the principle of work and energy, find N the number of revolutions the drum turns before it stops. b) Using the principle of angular impulse and momentum, find the time Δt required for the drum to stop.



Answers:

a) $N \approx 15.3$ revolutions; b) $\Delta t \approx 4.8\ \text{(sec)}$

5. The figure represents a rack and pinion gearing system. Rack *R* is pulled to the right by the force $P = 200\ \text{(N)}$, causing the gear to rotate about its mass center *O*. The gear has mass $m = 30\ \text{(kg)}$, radius $r = 0.15\ \text{(m)}$, and radius of gyration $k_O = 0.125\ \text{(m)}$. Rack *R* has mass $m_R = 20\ \text{(kg)}$ and slides freely on the smooth horizontal surface. The gear rolls without slipping on the rack at *C*. Using the principle of impulse and momentum, find: a) Δt the time it takes the gear to reach an angular velocity of $20\underset{\sim}{k}\ \text{(rad/s)}$, and b) f the horizontal force applied to the gear by the rack. The system starts from rest.



Answers:

a) $\Delta t \approx 0.613\ \text{(sec)}$; b) $f \approx 102\ \text{(N)}$