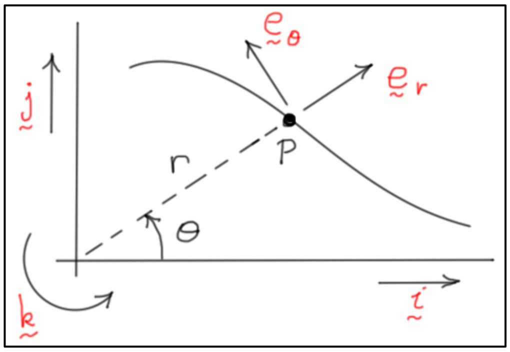
ME 5550 Intermediate Dynamics
Exercises #1

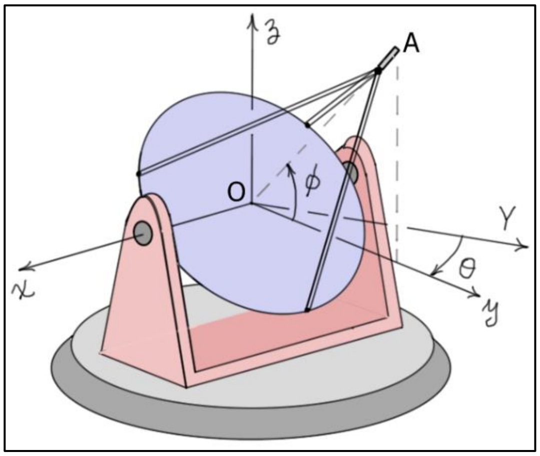
1) Radial and Transverse Components: The diagram shows two reference frames and a point P. The unit vector set $(\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k})$ defines the fixed frame R, and the unit vector set $(\underset{\sim}{e}_r, \underset{\sim}{e}_\theta, \underset{\sim}{k})$ defines the rotating frame E. Given that the position vector of a point P is $\underset{\sim}{r}_P = r\,\underset{\sim}{e}_r$ and that ${}^R\underset{\sim}{\omega}_E = \dot{\theta}\,\underset{\sim}{k}$, show that the velocity and acceleration of P can be written as



$$\underset{\sim}{v}_P = \dot{r}\,\underset{\sim}{e}_r + r\dot{\theta}\,\underset{\sim}{e}_\theta$$

$$\underset{\sim}{a}_P = (\ddot{r} - r\dot{\theta}^2)\,\underset{\sim}{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\underset{\sim}{e}_\theta$$

2) The antenna system shown has two components, the base B and the antenna dish D. The distance from O to A is L. The axes (x, y, z) rotate with the base B. At any instant, the angle between the y-axis and the fixed Y-axis is given by the angle θ, and the angle between OA and the y-axis is given by the angle ϕ. Assuming you know values for θ, ϕ, and their first and second time derivatives, a) find ${}^R\underset{\sim}{\omega}_D$ and ${}^R\underset{\sim}{\alpha}_D$ the angular velocity and angular acceleration of the dish D in a fixed reference frame R; and b) find ${}^R\underset{\sim}{v}_A$ and ${}^R\underset{\sim}{a}_A$ the velocity and acceleration of point A in a fixed frame R using *direct differentiation*. (The figure is based on a figure in Hibbeler, *Engineering Mechanics*, 1998)



3) The system shown has three components, a vertical column C, a horizontal arm M, and a disk D. The disk has radius r and rotates relative to the arm at a rate of ω_3 (rad/sec). The arm has length L and rotates relative to the column at a rate of ω_1 (rad/sec). The column rotates at a rate of ω_2 (rad/sec). Given that ω_1, ω_2, and ω_3 are *not* constant, a) find ${}^R\underset{\sim}{\omega}_D$ and ${}^R\underset{\sim}{\alpha}_D$ the angular velocity and angular acceleration of the disk D in the fixed reference frame R; and b) find ${}^R\underset{\sim}{v}_A$ and ${}^R\underset{\sim}{a}_A$ the velocity and acceleration of point A in a fixed frame R using *direct differentiation*. <u>Hint</u>: You will find it necessary to define an angle ϕ between the plane of the disk and the $(\underset{\sim}{e}_1, \underset{\sim}{e}_2)$ plane.