

ME 5550 Intermediate Dynamics

Exercises #1 Answers

1. Answers given.

2. Reference frame $B: (\underline{e}_1, \underline{e}_2, \underline{k})$ along the (x, y, z) axes.

$$a) {}^R \underline{\omega}_D = \dot{\phi} \underline{e}_1 - \dot{\theta} \underline{k}$$

$${}^R \underline{\alpha}_D = \ddot{\phi} \underline{e}_1 - \dot{\theta} \dot{\phi} \underline{e}_2 - \ddot{\theta} \underline{k}$$

$$b) \underline{v}_A = L(\dot{\theta} C_\phi \underline{e}_1 - \dot{\phi} S_\phi \underline{e}_2 + \dot{\phi} C_\phi \underline{k})$$

$$\underline{a}_A = L\left(\left(\ddot{\theta} C_\phi - 2\dot{\theta} \dot{\phi} S_\phi\right) \underline{e}_1 - \left(\ddot{\phi} S_\phi + \dot{\phi}^2 C_\phi + \dot{\theta}^2 C_\phi\right) \underline{e}_2 + \left(\ddot{\phi} C_\phi - \dot{\phi}^2 S_\phi\right) \underline{k}\right)$$

3. Reference frames: R (fixed frame), $C: (\underline{e}_1, \underline{e}_2, \underline{e}_3)$, and $M: (\underline{n}_1, \underline{n}_2, \underline{n}_3)$

(\underline{n}_3 is normal to the disk, $\underline{n}_2 = \underline{n}_3 \times \underline{e}_1$, and ϕ is the angle between the disk and the $(\underline{e}_1, \underline{e}_2)$ plane.)

$$a) {}^R \underline{\omega}_D = \omega_1 \underline{e}_1 + \omega_2 C_\phi \underline{n}_2 + (\omega_3 - \omega_2 S_\phi) \underline{n}_3$$

$${}^R \underline{\alpha}_D = (\dot{\omega}_1 + \omega_2 \omega_3 C_\phi) \underline{e}_1 + (\dot{\omega}_2 C_\phi - \omega_1 \omega_2 S_\phi - \omega_1 \omega_3) \underline{n}_2 + (\dot{\omega}_3 - \dot{\omega}_2 S_\phi - \omega_1 \omega_2 C_\phi) \underline{n}_3$$

$$b) {}^R \underline{v}_A = r(\omega_2 S_\theta S_\phi - \omega_3 S_\theta) \underline{e}_1 + (r \omega_3 C_\theta - (L + r C_\theta) \omega_2 S_\phi) \underline{n}_2 + (r \omega_1 S_\theta - (L + r C_\theta) \omega_2 C_\phi) \underline{n}_3$$

$${}^R \underline{a}_A = (r \dot{\omega}_2 S_\theta S_\phi - r \dot{\omega}_3 S_\theta - r \omega_3^2 C_\theta - (L + r C_\theta) \omega_2^2 + 2r \omega_1 \omega_2 S_\theta C_\phi + 2r \omega_2 \omega_3 S_\phi C_\theta) \underline{e}_1 + (r \dot{\omega}_3 C_\theta - (L + r C_\theta) \dot{\omega}_2 S_\phi - r \omega_1^2 S_\theta - r \omega_2^2 S_\theta S_\phi^2 - r \omega_3^2 S_\theta + 2r \omega_2 \omega_3 S_\theta S_\phi) \underline{n}_2 + (r \dot{\omega}_1 S_\theta - (L + r C_\theta) \dot{\omega}_2 C_\phi - r \omega_2^2 S_\theta S_\phi C_\phi + 2r \omega_2 \omega_3 S_\theta C_\phi + 2r \omega_1 \omega_3 C_\theta) \underline{n}_3$$