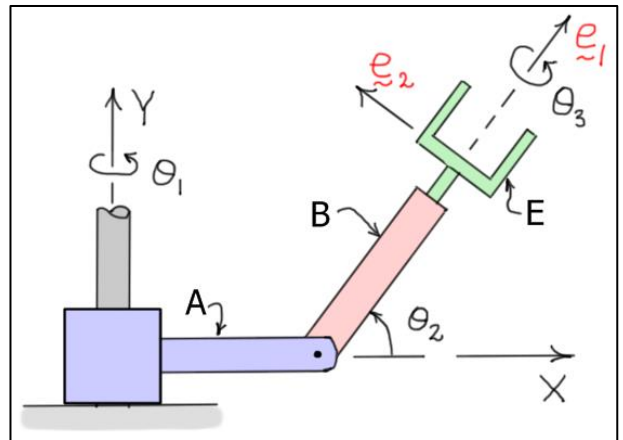


ME 5550 Intermediate Dynamics

Exercises #5

1. The system shown consists of three components, the arms A and B and the end-effector E . The orientation of E relative to a fixed frame is described by the three angles shown. Note that the sequence of rotations θ_1 , θ_2 , and θ_3 is a 2-3-1 body-fixed rotation sequence. Complete the following: a) Derive the transformation matrix $[R]$ that relates the unit vectors $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ (fixed in E) to the unit vectors $(\underline{N}_1, \underline{N}_2, \underline{N}_3)$ of the fixed frame.



b) Find the \underline{e}_i components of ${}^R\omega_E$ the angular velocity of E in R . c) Invert the equations from part (b) to solve for $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\theta}_3$ in terms of the angular velocity components. d) Find ${}^R\alpha_E$ the angular acceleration of E relative to the fixed frame. Note: In all parts of this problem, assume the angles θ_1 , θ_2 , θ_3 and their time-derivatives are all *nonzero*.

2. For the yoke-and-spider *universal joint* shown below, the unit vectors fixed in the shaft B : $(\underline{e}_1, \underline{e}_2, \underline{e}_3)$ are oriented relative to the fixed-frame R : $(\underline{N}_1, \underline{N}_2, \underline{N}_3)$ using a 1-2-3 body-fixed rotation sequence. The figure shows the configuration where all the angles are *zero*. In its final configuration, the shaft B is aligned with the unit vector \underline{n} so that $\underline{e}_1 = C_\phi \underline{N}_1 + S_\phi \underline{N}_3$. Using results in the notes for a 1-2-3 body-fixed rotation sequence, complete the following: a) Show that $C_2 C_3 = C_\phi$ and $S_3 = S_1 S_\phi$. b) Show that $\omega_B = \left(C_\phi / (1 - S_1^2 S_\phi^2) \right) \dot{\theta}_1$ where ω_B is the angular speed of shaft B about the \underline{n} direction. c) Plot the ratio $(\omega_B / \dot{\theta}_1)$ versus θ_1 for one complete revolution of shaft A for $\phi = 20^\circ$ and $\phi = 40^\circ$.

