

ME 5550 Intermediate Dynamics
Exercises #10 Answers

1. $\theta_{eq} = 0$ and $\theta_{eq} = 45^0$

2.
$$\begin{bmatrix} (m_1 + m_2) & \frac{1}{2} m_2 \ell \\ \frac{1}{2} m_2 \ell & \frac{1}{3} m_2 \ell^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & \frac{1}{2} m_2 g \ell \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix}$$

$$\{\omega_n\} \cong \begin{Bmatrix} 4.89 \\ 29.44 \end{Bmatrix} \text{ (r/s)} \cong \begin{Bmatrix} 0.7784 \\ 4.686 \end{Bmatrix} \text{ (Hz)}$$

Mode 1: $\underline{u}_1 = \begin{Bmatrix} 1 \\ 77.8 \end{Bmatrix}$

Mode 2: $\underline{u}_2 = \begin{Bmatrix} 1 \\ -0.7715 \end{Bmatrix}$

3. $\theta_{eq} = 0$ and $\theta_{eq} = 90^0$

$\theta_{eq} = 0 \Rightarrow \Delta\ddot{\theta} - \Omega^2(\Delta\theta) = 0$ (unstable equilibrium)

$\theta_{eq} = 90^0 \Rightarrow \Delta\ddot{\theta} + \Omega^2(\Delta\theta) = 0$ (stable equilibrium)

$M_\phi = \frac{1}{6} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$

4. $\theta_{eq} = 58.7^0 \Rightarrow \Delta\ddot{\theta} + (133.52)\Delta\theta = 0$ (stable equilibrium)

$M_\phi = \frac{2}{3} m \ell^2 S_\theta C_\theta \Omega \dot{\theta}$