

## ME 5550 Intermediate Dynamics

### Exercises #11 Answers

1. Lagrangian:  $L = \frac{1}{8}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}m_p\dot{y}^2 + \frac{1}{24}m\ell^2\dot{\theta}^2 - m_p g y - \frac{1}{2}m g y - \frac{1}{2}k x^2$

Three differential/algebraic equations from Lagrange's equations:

$$\begin{aligned}\frac{1}{4}m\ddot{x} + c\dot{x} + kx &= \lambda_1 \\ (m_p + \frac{1}{4}m)\ddot{y} + (m_p + \frac{1}{2}m)g &= \lambda_2 - F(t) \\ \left(\frac{1}{12}m\ell^2\right)\ddot{\theta} &= (-\ell C_\theta)\lambda_1 + (\ell S_\theta)\lambda_2\end{aligned}$$

Two differentiated constraint equations:

$$\begin{aligned}\ddot{x} - (\ell C_\theta)\ddot{\theta} + (\ell S_\theta)\dot{\theta}^2 &= 0 \\ \ddot{y} + (\ell S_\theta)\ddot{\theta} + (\ell C_\theta)\dot{\theta}^2 &= 0\end{aligned}$$

2. Lagrangian:  $L = \frac{1}{2}m_1\dot{x}^2 + \frac{1}{2}m_2\left[(\dot{x} + \dot{x}_G)^2 + \dot{y}_G^2\right] + \frac{1}{24}m_2\ell^2\dot{\theta}^2 - \frac{1}{2}kx^2 + m_2gy_G$

Four differential/algebraic equations from Lagrange's equations:

$$\begin{aligned}(m_1 + m_2)\ddot{x} + m_2\ddot{x}_G + c\dot{x} + kx &= F(t) \\ m_2\ddot{x} + m_2\ddot{x}_G &= \lambda_1 \\ m_2\ddot{y}_G - m_2g &= \lambda_2 \\ \left(\frac{1}{12}m_2\ell^2\right)\ddot{\theta} &= \left(-\frac{1}{2}\ell C_\theta\right)\lambda_1 + \left(\frac{1}{2}\ell S_\theta\right)\lambda_2\end{aligned}$$

Two differentiated constraint equations:

$$\begin{aligned}\ddot{x}_G - \left(\frac{1}{2}\ell C_\theta\right)\ddot{\theta} + \left(\frac{1}{2}\ell S_\theta\right)\dot{\theta}^2 &= 0 \\ \ddot{y}_G + \left(\frac{1}{2}\ell S_\theta\right)\ddot{\theta} + \left(\frac{1}{2}\ell C_\theta\right)\dot{\theta}^2 &= 0\end{aligned}$$