

ME 6590 Multibody Dynamics

Exercises #1 Answers

1. a) The column rotates about the Y axis, and the disk is oriented with a body-fixed 2-1-3 rotation sequence.

$$\begin{aligned} \{r_P\} &= [R_C]^T \{L'\} + [R_D]^T \{r'\} \\ &= \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C_1 C_3 + S_1 S_2 S_3 & -C_1 S_3 + S_1 S_2 C_3 & S_1 C_2 \\ C_2 S_3 & C_2 C_3 & -S_2 \\ -S_1 C_3 + C_1 S_2 S_3 & S_1 S_3 + C_1 S_2 C_3 & C_1 C_2 \end{bmatrix} \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

- b) ω'_{Di} ($i=1,2,3$) are the body-fixed components of the angular velocity of the disk.

$$\begin{aligned} \{{}^R v_P\} &= [R_C]^T [\tilde{\omega}'_C] \{L'\} + [R_D]^T [\tilde{\omega}'_D] \{r'\} = \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} \\ &+ \begin{bmatrix} C_1 C_3 + S_1 S_2 S_3 & -C_1 S_3 + S_1 S_2 C_3 & S_1 C_2 \\ C_2 S_3 & C_2 C_3 & -S_2 \\ -S_1 C_3 + C_1 S_2 S_3 & S_1 S_3 + C_1 S_2 C_3 & C_1 C_2 \end{bmatrix} \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} \omega'_{D1} \\ \omega'_{D2} \\ \omega'_{D3} \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 C_2 S_3 + \dot{\theta}_2 C_3 \\ \dot{\theta}_1 C_2 C_3 - \dot{\theta}_2 S_3 \\ -\dot{\theta}_1 S_2 + \dot{\theta}_3 \end{bmatrix} \quad (\text{for a 2-1-3 body-fixed rotation sequence})$$

- c) α'_{Di} ($i=1,2,3$) are the body-fixed components of the angular acceleration of the disk.

$$\begin{aligned} \{{}^R a_P\} &= [R_C]^T \left([\tilde{\alpha}'_C] + [\tilde{\omega}'_C][\tilde{\omega}'_C] \right) \{L'\} + [R_D]^T \left([\tilde{\alpha}'_D] + [\tilde{\omega}'_D][\tilde{\omega}'_D] \right) \{r'\} \\ &= \begin{bmatrix} C_1 & 0 & S_1 \\ 0 & 1 & 0 \\ -S_1 & 0 & C_1 \end{bmatrix} \left(\begin{bmatrix} 0 & 0 & \ddot{\theta}_1 \\ 0 & 0 & 0 \\ -\ddot{\theta}_1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \right) \begin{bmatrix} L \\ 0 \\ 0 \end{bmatrix} + \\ &\begin{bmatrix} C_1 C_3 + S_1 S_2 S_3 & -C_1 S_3 + S_1 S_2 C_3 & S_1 C_2 \\ C_2 S_3 & C_2 C_3 & -S_2 \\ -S_1 C_3 + C_1 S_2 S_3 & S_1 S_3 + C_1 S_2 C_3 & C_1 C_2 \end{bmatrix} \left(\begin{bmatrix} 0 & -\alpha'_{D3} & \alpha'_{D2} \\ \alpha'_{D3} & 0 & -\alpha'_{D1} \\ -\alpha'_{D2} & \alpha'_{D1} & 0 \end{bmatrix} + \right. \\ &\left. \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \begin{bmatrix} 0 & -\omega'_{D3} & \omega'_{D2} \\ \omega'_{D3} & 0 & -\omega'_{D1} \\ -\omega'_{D2} & \omega'_{D1} & 0 \end{bmatrix} \right) \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

- d) The column vectors below show the \underline{N}_1 , \underline{N}_2 , and \underline{N}_3 components of the position, velocity, and acceleration vectors of P .

$$\left\{ r_P \right\} = \begin{Bmatrix} LC_1 + r(C_1C_3 + S_1S_2S_3) \\ rC_2S_3 \\ -LS_1 + r(-S_1C_3 + C_1S_2S_3) \end{Bmatrix}$$

$$\left\{ {}^R v_P \right\} = \begin{Bmatrix} -LS_1\dot{\theta}_1 + r(-C_1S_3 + S_1S_2C_3)\omega'_{D3} - rS_1C_2\omega'_{D2} \\ r(C_2C_3\omega'_{D3} + S_2\omega'_{D2}) \\ -LC_1\dot{\theta}_1 + r(S_1S_3 + C_1S_2C_3)\omega'_{D3} - rC_1C_2\omega'_{D2} \end{Bmatrix}$$

$$\left\{ {}^R a_P \right\} = L \begin{Bmatrix} -(C_1\ddot{\theta}_1 + S_1\ddot{\theta}_1) \\ 0 \\ (S_1\dot{\theta}_1^2 - C_1\ddot{\theta}_1) \end{Bmatrix} + r \begin{Bmatrix} -(C_1C_3 + S_1S_2S_3)(\omega'_{D2}{}^2 + \omega'_{D3}{}^2) + (-C_1S_3 + S_1S_2C_3)(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}) + S_1C_2(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}) \\ -C_2S_3(\omega'_{D2}{}^2 + \omega'_{D3}{}^2) + C_2C_3(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}) - S_2(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}) \\ (S_1C_3 - C_1S_2S_3)(\omega'_{D2}{}^2 + \omega'_{D3}{}^2) + (S_1S_3 + C_1S_2C_3)(\alpha'_{D3} + \omega'_{D1}\omega'_{D2}) + C_1C_2(-\alpha'_{D2} + \omega'_{D1}\omega'_{D3}) \end{Bmatrix}$$

2. a) $\varepsilon_1 = 0.142857$; $\varepsilon_2 = -0.214286$; $\varepsilon_3 = 0.428571$; $\varepsilon_4 = 0.866025$

b) $[R] = \begin{bmatrix} 0.540816 & 0.681083 & 0.493603 \\ -0.803532 & 0.591837 & 0.063762 \\ -0.248705 & -0.431109 & 0.867347 \end{bmatrix}$

c) $\underline{e}_1 = 0.540816 \underline{N}_1 + 0.681083 \underline{N}_2 + 0.493603 \underline{N}_3$

$$\underline{e}_2 = -0.803532 \underline{N}_1 + 0.591837 \underline{N}_2 + 0.063762 \underline{N}_3$$

$$\underline{e}_3 = -0.248705 \underline{N}_1 - 0.431109 \underline{N}_2 + 0.867347 \underline{N}_3$$

3. a) $\varepsilon_1 = 0.022260$; $\varepsilon_2 = -0.439680$; $\varepsilon_3 = 0.531976$; $\varepsilon_4 = 0.723317$

b) $\omega'_1 = -0.375$ (rad/s); $\omega'_2 = 1.51554$ (rad/s); $\omega'_3 = 1.75$ (rad/s)

c) $\dot{\varepsilon}_1 = 0.652214$; $\dot{\varepsilon}_2 = 0.667333$; $\dot{\varepsilon}_3 = 698475$; $\dot{\varepsilon}_4 = -0.128128$