ME 6590 Multibody Dynamics
Exercises #3 Answers

1. $\left\{a_{G_3}\right\} = \left\{\ddot{s}_1\right\} + \sum_{i=2}^{3}\left[v_{G_3,\dot{s}'_i}\right]\left\{\ddot{s}'_i\right\} + \sum_{i=2}^{3}\left[\dot{v}_{G_3,\dot{s}'_i}\right]\left\{\dot{s}'_i\right\} + \sum_{i=1}^{3}\left[v_{G_3,\dot{\theta}_i}\right]\left\{\ddot{\theta}_i\right\} + \sum_{i=1}^{3}\left[\dot{v}_{G_3,\dot{\theta}_i}\right]\left\{\dot{\theta}_i\right\}$

   Here, the partial velocities are as given in Exercises #2. The derivatives of the partial velocities are

$$\left[\dot{v}_{G_2,\dot{s}'_2}\right] = \left[\dot{R}_{B_1}\right]^T = \left[R_{B_1}\right]^T\left[\tilde{\omega}'_{B_1}\right]$$

$$\begin{aligned}\left[\dot{v}_{G_2,\dot{\theta}_1}\right] = &-\left[R_{B_1}\right]^T\left[\tilde{\omega}'_{B_1}\right]\left(\left[\tilde{q}'_2\right]+\left[\tilde{s}'_2\right]\right)\left[\omega'_{B_1,\dot{\theta}_1}\right] - \left[R_{B_1}\right]^T\left[\dot{\tilde{s}}'_2\right]\left[\omega'_{B_1,\dot{\theta}_1}\right] \\ &-\left[R_{B_1}\right]^T\left(\left[\tilde{q}'_2\right]+\left[\tilde{s}'_2\right]\right)\left[\dot{\omega}'_{B_1,\dot{\theta}_1}\right]\end{aligned}$$

$$\left[\dot{v}_{G_2,\dot{\theta}_2}\right] = -\left[R_{B_2}\right]^T\left[\tilde{\omega}'_{B_2}\right]\left[\tilde{r}'_2\right]\left[\omega'_{B_2,\dot{\theta}_2}\right] - \left[R_{B_2}\right]^T\left[\tilde{r}'_2\right]\left[\dot{\omega}'_{B_2,\dot{\theta}_2}\right]$$

2. $\left\{a_{G_3}\right\} = \left\{\ddot{s}_1\right\} + \sum_{i=2}^{3}\left[v_{G_3,\dot{s}'_i}\right]\left\{\ddot{s}'_i\right\} + \sum_{i=2}^{3}\left[\dot{v}_{G_3,\dot{s}'_i}\right]\left\{\dot{s}'_i\right\} + \sum_{i=1}^{3}\left[v_{G_3,\dot{\theta}_i}\right]\left\{\ddot{\theta}_i\right\} + \sum_{i=1}^{3}\left[\dot{v}_{G_3,\dot{\theta}_i}\right]\left\{\dot{\theta}_i\right\}$

   where the partial velocities are as given in Exercises #2. The derivatives of the partial velocities are

$$\left[\dot{v}_{G_3,\dot{s}'_2}\right] = \left[\dot{R}_{B_1}\right]^T = \left[R_{B_1}\right]^T\left[\tilde{\omega}'_{B_1}\right]$$

$$\left[\dot{v}_{G_3,\dot{s}'_3}\right] = \left[\dot{R}_{B_2}\right]^T = \left[R_{B_2}\right]^T\left[\tilde{\omega}'_{B_2}\right]$$

$$\begin{aligned}\left[\dot{v}_{G_3,\dot{\theta}_1}\right] = &-\left[R_{B_1}\right]^T\left[\tilde{\omega}'_{B_1}\right]\left(\left[\tilde{q}'_2\right]+\left[\tilde{s}'_2\right]\right)\left[\omega'_{B_1,\dot{\theta}_1}\right] - \left[R_{B_1}\right]^T\left[\dot{\tilde{s}}'_2\right]\left[\omega'_{B_1,\dot{\theta}_1}\right] \\ &-\left[R_{B_1}\right]^T\left(\left[\tilde{q}'_2\right]+\left[\tilde{s}'_2\right]\right)\left[\dot{\omega}'_{B_1,\dot{\theta}_1}\right]\end{aligned}$$

$$\begin{aligned}\left[\dot{v}_{G_3,\dot{\theta}_2}\right] = &-\left[R_{B_2}\right]^T\left[\tilde{\omega}'_{B_2}\right]\left(\left[\tilde{q}'_3\right]+\left[\tilde{s}'_3\right]\right)\left[\omega'_{B_2,\dot{\theta}_2}\right] - \left[R_{B_2}\right]^T\left[\dot{\tilde{s}}'_3\right]\left[\omega'_{B_2,\dot{\theta}_2}\right] \\ &-\left[R_{B_2}\right]^T\left(\left[\tilde{q}'_3\right]+\left[\tilde{s}'_3\right]\right)\left[\dot{\omega}'_{B_2,\dot{\theta}_2}\right]\end{aligned}$$

$$\left[\dot{v}_{G_3,\dot{\theta}_3}\right] = -\left[R_{B_3}\right]^T\left[\tilde{\omega}'_{B_3}\right]\left[\tilde{r}'_3\right]\left[\omega'_{B_3,\dot{\theta}_3}\right] - \left[R_{B_3}\right]^T\left[\tilde{r}'_3\right]\left[\dot{\omega}'_{B_3,\dot{\theta}_3}\right]$$