

# ME 6590 Multibody Dynamics

## Exercises #4 Answers

1. a) Relative coordinates:

Translation constraints (6 equations)

$$\boxed{\{s_1\} = \{0\}_{3 \times 1}} \quad \text{and} \quad \boxed{\{s'_2\} = \{0\}_{3 \times 1}}$$

Rotation constraints (3 equations) with

$$\boxed{{}^R \omega_{L_1} = \hat{\omega}_{L_1} = \sum_{i=1}^3 \hat{\omega}'_{li} \underline{e}_{li}}, \quad \boxed{{}^{B_1} \omega_{L_2} = \hat{\omega}_{L_2} = \sum_{i=1}^3 \hat{\omega}'_{2i} \underline{e}_{2i}}$$

$$\boxed{[0 \ 1 \ 0] [R_{L_1}]^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}'_{11} \\ \hat{\omega}'_{12} \\ \hat{\omega}'_{13} \end{Bmatrix}} \quad \text{and} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}'_{21} \\ \hat{\omega}'_{22} \\ \hat{\omega}'_{23} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

b) Absolute coordinates:

Translation constraints (6 equations)

$$\boxed{\{p_{G_1}\} + \frac{1}{2} \ell_1 [R_{L_1}]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}} \quad \boxed{\{p_{G_2}\} - \{p_{G_1}\} + \frac{1}{2} \ell_1 [R_{L_1}]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} + \frac{1}{2} \ell_2 [R_{L_2}]^T \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}}$$

Rotation constraints (3 equations) with

$$\boxed{{}^R \omega_{L_1} = \sum_{i=1}^3 \omega'_{li} \underline{e}_{li}} \quad \text{and} \quad \boxed{{}^R \omega_{L_2} = \sum_{i=1}^3 \omega'_{2i} \underline{e}_{2i}}$$

$$\boxed{[0 \ 1 \ 0] [R_{L_1}]^T \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \omega'_{11} \\ \omega'_{12} \\ \omega'_{13} \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \omega'_{21} \\ \omega'_{22} \\ \omega'_{23} \end{Bmatrix} - [R_{L_2}] [R_{L_1}]^T \begin{Bmatrix} \omega'_{11} \\ \omega'_{12} \\ \omega'_{13} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

2. a) Relative coordinates:

Translation constraints (8 equations)

$$\boxed{\{s_1\} = \{0\}_{3 \times 1}} \quad \boxed{\{s'_2\} = \{0\}_{3 \times 1}} \quad \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} s'_{31} \\ s'_{32} \\ s'_{33} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

Rotation constraints (6 eq.):

$$\boxed{{}^R \omega_B = \hat{\omega}_B = \sum_{i=1}^3 \hat{\omega}'_{Bi} \underline{e}_{Bi}} \quad \boxed{{}^B \omega_M = \hat{\omega}_M = \sum_{i=1}^3 \hat{\omega}'_{Mi} \underline{e}_{Mi}} \quad \boxed{{}^M \omega_E = \hat{\omega}_E = \sum_{i=1}^3 \hat{\omega}'_{Ei} \underline{e}_{Ei}}$$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \hat{\omega}'_{B1} \\ \hat{\omega}'_{B2} \\ \hat{\omega}'_{B3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} \hat{\omega}'_{M1} \\ \hat{\omega}'_{M2} \\ \hat{\omega}'_{M3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}} \quad \boxed{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \hat{\omega}'_{E1} \\ \hat{\omega}'_{E2} \\ \hat{\omega}'_{E3} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}}$$

b) Absolute Coordinates:

Translation constraints (8 equations)

$$\left\{ p_{G_B} \right\} - \left\{ \begin{matrix} 0 \\ \ell_1 \\ 0 \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right\} \quad \left\{ p_{G_M} \right\} - \left\{ p_{G_B} \right\} - \left\{ \begin{matrix} 0 \\ \ell_2 \\ 0 \end{matrix} \right\} - [R_M]^T \left\{ \begin{matrix} \ell_3 \\ 0 \\ 0 \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right\}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left( [R_M] \left\{ p_{G_E} \right\} - [R_M] \left\{ p_{G_M} \right\} - (\ell_4 + \ell_5) \left\{ \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \right\} \right) = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

Rotation constraints (6 equations):

$${}^R \omega_B = \sum_{i=1}^3 \omega'_{Bi} \underline{e}_{Bi}$$

$${}^R \omega_M = \sum_{i=1}^3 \omega'_{Mi} \underline{e}_{Mi}$$

$${}^R \omega_E = \sum_{i=1}^3 \omega'_{Ei} \underline{e}_{Ei}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left\{ \begin{matrix} \omega'_{B1} \\ \omega'_{B2} \\ \omega'_{B3} \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \left( \left\{ \begin{matrix} \omega'_{M1} \\ \omega'_{M2} \\ \omega'_{M3} \end{matrix} \right\} - [R_M] [R_B]^T \left\{ \begin{matrix} \omega'_{B1} \\ \omega'_{B2} \\ \omega'_{B3} \end{matrix} \right\} \right) = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left( \left\{ \begin{matrix} \omega'_{E1} \\ \omega'_{E2} \\ \omega'_{E3} \end{matrix} \right\} - [R_E] [R_M]^T \left\{ \begin{matrix} \omega'_{M1} \\ \omega'_{M2} \\ \omega'_{M3} \end{matrix} \right\} \right) = \left\{ \begin{matrix} 0 \\ 0 \end{matrix} \right\}$$

