

## ME 6590 Multibody Dynamics

### Exercises #6 Answers

1. a)  $\left(\frac{1}{3}mL^2\right)\ddot{\theta} + \frac{1}{2}mgL\sin(\theta) = 0$

b) Three differential/algebraic equations from Lagrange's equations

$$m\ddot{x} = \lambda_1$$

$$m\ddot{y} + mg = \lambda_2$$

$$\left(\frac{1}{12}mL^2\right)\ddot{\theta} = \left(-\frac{1}{2}L\cos(\theta)\right)\lambda_1 + \left(-\frac{1}{2}L\sin(\theta)\right)\lambda_2$$

Two differentiated constraint equations

$$\ddot{x} - \left(\frac{1}{2}L\cos(\theta)\right)\ddot{\theta} + \left(\frac{1}{2}L\sin(\theta)\right)\dot{\theta}^2 = 0$$

$$\ddot{y} - \left(\frac{1}{2}L\sin(\theta)\right)\ddot{\theta} - \left(\frac{1}{2}L\cos(\theta)\right)\dot{\theta}^2 = 0$$

2. a)  $\left(\frac{4}{3}mL^2\right)\ddot{\theta}_1 + \left(\frac{1}{2}mL^2\cos(\theta_2 - \theta_1)\right)\ddot{\theta}_2 - \left(\frac{1}{2}mL^2\sin(\theta_2 - \theta_1)\right)\dot{\theta}_2^2 + \left(\frac{3}{2}mgL\right)\sin(\theta_1) = 0$

$$\left(\frac{1}{2}mL^2\cos(\theta_2 - \theta_1)\right)\ddot{\theta}_1 + \left(\frac{1}{3}mL^2\right)\ddot{\theta}_2 + \left(\frac{1}{2}mL^2\sin(\theta_2 - \theta_1)\right)\dot{\theta}_1^2 + \left(\frac{1}{2}mgL\right)\sin(\theta_2) = 0$$

b) Six differential/algebraic equations from Lagrange's equations

$$m\ddot{x}_1 = \lambda_1$$

$$m\ddot{y}_1 + mg = \lambda_2$$

$$\left(\frac{1}{12}mL^2\right)\ddot{\theta}_1 = \left(-\frac{1}{2}L\cos(\theta_1)\right)\lambda_1 + \left(-\frac{1}{2}L\sin(\theta_1)\right)\lambda_2 + \left(-L\cos(\theta_1)\right)\lambda_3 + \left(-L\sin(\theta_1)\right)\lambda_4$$

$$m\ddot{x}_2 = \lambda_3$$

$$m\ddot{y}_2 + mg = \lambda_4$$

$$\left(\frac{1}{12}mL^2\right)\ddot{\theta}_2 = \left(-\frac{1}{2}L\cos(\theta_2)\right)\lambda_3 + \left(-\frac{1}{2}L\sin(\theta_2)\right)\lambda_4$$

Four differentiated constraint equations

$$\ddot{x}_1 + \left(-\frac{1}{2}L\cos(\theta_1)\right)\ddot{\theta}_1 + \left(\frac{1}{2}L\sin(\theta_1)\right)\dot{\theta}_1^2 = 0$$

$$\ddot{y}_1 + \left(-\frac{1}{2}L\sin(\theta_1)\right)\ddot{\theta}_1 - \left(\frac{1}{2}L\cos(\theta_1)\right)\dot{\theta}_1^2 = 0$$

$$\ddot{x}_2 + \left(-L\cos(\theta_1)\right)\ddot{\theta}_1 + \left(L\sin(\theta_1)\right)\dot{\theta}_1^2 + \left(-\frac{1}{2}L\cos(\theta_2)\right)\ddot{\theta}_2 + \left(\frac{1}{2}L\sin(\theta_2)\right)\dot{\theta}_2^2 = 0$$

$$\ddot{y}_2 + \left(-L\sin(\theta_1)\right)\ddot{\theta}_1 - \left(L\cos(\theta_1)\right)\dot{\theta}_1^2 + \left(-\frac{1}{2}L\sin(\theta_2)\right)\ddot{\theta}_2 - \left(\frac{1}{2}L\cos(\theta_2)\right)\dot{\theta}_2^2 = 0$$

3. a) Two differential/algebraic equations from Lagrange's equations

$$\left(\frac{4}{3}mL^2\right)\ddot{\theta}_1 + \left(\frac{1}{2}mL^2\cos(\theta_2 - \theta_1)\right)\ddot{\theta}_2 - \left(\frac{1}{2}mL^2\sin(\theta_2 - \theta_1)\right)\dot{\theta}_2^2 + \left(\frac{3}{2}mgL\right)\sin(\theta_1) = M + \lambda_1$$

$$\left(\frac{1}{2}mL^2\cos(\theta_2 - \theta_1)\right)\ddot{\theta}_1 + \left(\frac{1}{3}mL^2\right)\ddot{\theta}_2 + \left(\frac{1}{2}mL^2\sin(\theta_2 - \theta_1)\right)\dot{\theta}_1^2 + \left(\frac{1}{2}mgL\right)\sin(\theta_2) = \lambda_1$$

One differentiated constraint equation

$$\dot{\theta}_1 + \dot{\theta}_2 = 0$$

b) Six differential/algebraic equations from Lagrange's equations

$$m\ddot{x}_1 = \lambda_1$$

$$m\ddot{y}_1 + mg = \lambda_2$$

$$\left(\frac{1}{12}mL^2\right)\ddot{\theta}_1 = M + \left(-\frac{L}{2}\cos(\theta_1)\right)\lambda_1 + \left(-\frac{L}{2}\sin(\theta_1)\right)\lambda_2 + \left(-L\cos(\theta_1)\right)\lambda_3 + \left(-L\sin(\theta_1)\right)\lambda_4 + \lambda_5$$

$$m\ddot{x}_2 = \lambda_3$$

$$m\ddot{y}_2 + mg = \lambda_4$$

$$\left(\frac{1}{12}mL^2\right)\ddot{\theta}_2 = \left(-\frac{L}{2}\cos(\theta_2)\right)\lambda_3 + \left(-\frac{L}{2}\sin(\theta_2)\right)\lambda_4 + \lambda_5$$

Five differentiated constraint equations

$$\dot{x}_1 + \left(-\frac{1}{2}L\cos(\theta_1)\right)\dot{\theta}_1 + \left(\frac{1}{2}L\sin(\theta_1)\right)\dot{\theta}_1^2 = 0$$

$$\dot{y}_1 + \left(-\frac{1}{2}L\sin(\theta_1)\right)\dot{\theta}_1 - \left(\frac{1}{2}L\cos(\theta_1)\right)\dot{\theta}_1^2 = 0$$

$$\dot{x}_2 + \left(-L\cos(\theta_1)\right)\dot{\theta}_1 + \left(L\sin(\theta_1)\right)\dot{\theta}_1^2 + \left(-\frac{1}{2}L\cos(\theta_2)\right)\dot{\theta}_2 + \left(\frac{1}{2}L\sin(\theta_2)\right)\dot{\theta}_2^2 = 0$$

$$\dot{y}_2 + \left(-L\sin(\theta_1)\right)\dot{\theta}_1 - \left(L\cos(\theta_1)\right)\dot{\theta}_1^2 + \left(-\frac{1}{2}L\sin(\theta_2)\right)\dot{\theta}_2 - \left(\frac{1}{2}L\cos(\theta_2)\right)\dot{\theta}_2^2 = 0$$

$$\dot{\theta}_1 + \dot{\theta}_2 = 0$$