

## ME 6590 Multibody Dynamics

### Exercises #7 Answers (d'Alembert's Principle)

$$1. \left[ \frac{1}{3} m \ell^2 + m_p \ell^2 S_\theta^2 \right] \ddot{\theta} + (m_p \ell^2 S_\theta C_\theta) \dot{\theta}^2 + (c \ell^2 C_\theta^2) \dot{\theta} + k \ell^2 S_\theta C_\theta - \left[ m_p g + \frac{1}{2} m g + F(t) \right] \ell S_\theta = 0$$

$$2. (m_1 + m_2) \ddot{x} + \left( \frac{1}{2} m_2 \ell C_\theta \right) \ddot{\theta} - \left( \frac{1}{2} m_2 \ell S_\theta \right) \dot{\theta}^2 + c \dot{x} + kx = F(t) \\ \left( \frac{1}{3} m_2 \ell^2 \right) \ddot{\theta} + \left( \frac{1}{2} m_2 \ell C_\theta \right) \ddot{x} + \frac{1}{2} m_2 g \ell S_\theta = 0$$

$$3. \left[ \frac{1}{2} m_s r^2 + \frac{1}{12} m \ell^2 S_\theta^2 \right] \ddot{\phi} + \left( \frac{1}{6} m \ell^2 S_\theta C_\theta \right) \dot{\theta} \dot{\phi} = M_\phi \\ \left( \frac{1}{12} m \ell^2 \right) \ddot{\theta} - \left( \frac{1}{12} m \ell^2 S_\theta C_\theta \right) \dot{\phi}^2 = M_\theta$$

$$4. \left[ \frac{1}{2} m_d R^2 + m \left( b + \frac{1}{2} \ell S_\theta \right)^2 + \frac{1}{12} m \ell^2 S_\theta^2 \right] \ddot{\phi} + \left( m b \ell C_\theta + \frac{2}{3} m \ell^2 S_\theta C_\theta \right) \dot{\theta} \dot{\phi} = M_\phi \\ \left( \frac{1}{3} m \ell^2 \right) \ddot{\theta} - \left[ \frac{1}{2} m b \ell C_\theta + \frac{1}{3} m \ell^2 S_\theta C_\theta \right] \dot{\phi}^2 + c \dot{\theta} + k \theta + \frac{1}{2} m g \ell S_\theta = M_\theta$$