

ME 6590 Multibody Dynamics

Exercises #9 Answers

1. There are **four** equations for $(\phi, \theta, \omega'_1, \omega'_2)$:

$$\begin{cases} (m\ell^2/12)S_\theta\dot{\omega}'_1 + (m\ell^2/12)C_\theta\omega'_1\omega'_2 = -M_\phi \\ (m\ell^2/12)\dot{\omega}'_2 - (m\ell^2C_\theta/12S_\theta)\omega_1'^2 = M_\theta \end{cases}$$

with the kinematical differential equations

$$\begin{cases} \dot{\phi} = -\omega'_1/S_\theta \\ \dot{\theta} = \omega'_2 \end{cases}$$

2. There are **seven** equations for $(\omega'_1, \omega'_2, \omega'_3, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$:

$$\begin{cases} (I + mL^2)\dot{\omega}'_1 + (I_3 - I - mL^2)\omega'_2\omega'_3 = 2mgL(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) \\ (I + mL^2)\dot{\omega}'_2 + (I + mL^2 - I_3)\omega'_1\omega'_3 = 2mgL(\varepsilon_2\varepsilon_4 - \varepsilon_1\varepsilon_3) \\ \omega'_3 = \text{constant} \end{cases}$$

with the kinematical differential equations

$$\{\dot{\varepsilon}\}_{4 \times 1} = \frac{1}{2} \begin{bmatrix} \varepsilon_4 & -\varepsilon_3 & \varepsilon_2 & \varepsilon_1 \\ \varepsilon_3 & \varepsilon_4 & -\varepsilon_1 & \varepsilon_2 \\ -\varepsilon_2 & \varepsilon_1 & \varepsilon_4 & \varepsilon_3 \\ -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & \varepsilon_4 \end{bmatrix} \begin{Bmatrix} \omega'_1 \\ \omega'_2 \\ \omega'_3 \\ 0 \end{Bmatrix}$$

3. a) Partial velocities and partial angular velocities

$$\begin{aligned} \partial v_G / \partial \omega'_1 &= L e_3 / 2 & \partial v_G / \partial \omega'_2 &= 0 & \partial v_G / \partial \omega'_3 &= -L e_1 / 2 \\ \partial v_A / \partial \omega'_1 &= L e_3 & \partial v_A / \partial \omega'_2 &= 0 & \partial v_A / \partial \omega'_3 &= -L e_1 \\ \partial v_B / \partial \omega'_1 &= L e_3 / 2 & \partial v_B / \partial \omega'_2 &= -L e_3 / 2 & \partial v_B / \partial \omega'_3 &= (L/2)(-e_1 + e_2) \end{aligned}$$

b) Generalized forces

$$\begin{cases} F_{\omega'_1} = LF_{A3} + LF_B / 2 + mgLS_1C_2 \\ F_{\omega'_2} = -LF_B / 2 \\ F_{\omega'_3} = -LF_{A1} + mgL(C_1S_3 + S_1S_2C_3) \end{cases}$$

c)

$$\begin{cases} \left(\frac{7}{12}mL^2\right)\dot{\omega}'_1 + \left(\frac{7}{12}mL^2\right)\omega'_2\omega'_3 = F_{\omega'_1} \\ \left(\frac{1}{12}mL^2\right)\dot{\omega}'_2 - \left(\frac{1}{12}mL^2\right)\omega'_1\omega'_3 = F_{\omega'_2} \\ \left(\frac{2}{3}mL^2\right)\dot{\omega}'_3 - \left(\frac{1}{2}mL^2\right)\omega'_1\omega'_2 = F_{\omega'_3} \end{cases}$$

d)

$$\begin{cases} \dot{\theta}_1 = (\omega'_1C_3 - \omega'_2S_3)/C_2 \\ \dot{\theta}_2 = \omega'_1S_3 + \omega'_2C_3 \\ \dot{\theta}_3 = \omega'_3 + (-\omega'_1C_3 + \omega'_2S_3)S_2/C_2 \end{cases}$$