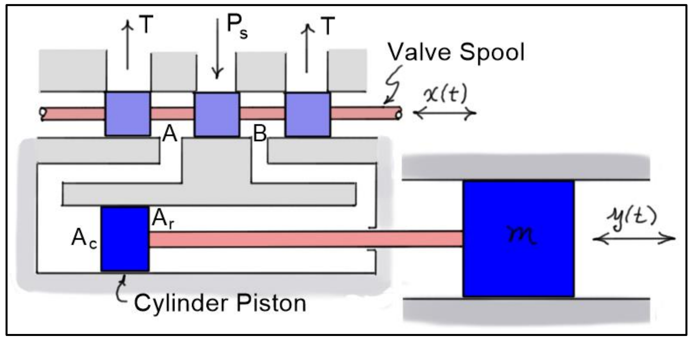
ME 4710 Motion and Control
Exercises #2 Basic System Modeling and Control

1. For the following problem, refer to the notes entitled "Hydraulic Positioning System I."



a) Together, equations (5) and (6) describe the extension of a hydraulic cylinder with an incompressible fluid. Using equation (7) (found by combining equations (5) and (6)), find the transfer function $\frac{Y}{X}(s)$. b) Together, equations (6) and (14) describe the extension of a pneumatic cylinder. Find the transfer functions $\frac{Y}{X}(s)$ and $\frac{P}{X}(s)$. c) Find $\dot{y}_{ss}$ the steady-state cylinder speed for parts (a) and (b) and find p_{ss} the steady-state pressure for part (b) given a unit step input $X(s)=\frac{1}{s}$.

Answers:

a) $\frac{Y}{X}(s)=\frac{A_c\left(k_x/k_p\right)}{s\left[ms+\left(b+(A_c^2/k_p)\right)\right]}$;

b) $\frac{Y}{X}(s)=\frac{A_c k_x}{s\left[m\left(A_cY_0/P_0\right)s^2+\left\{mk_p+b\left(A_cY_0/P_0\right)\right\}s+\left(bk_p+A_c^2)\right)\right]}$

$\frac{P}{X}(s)=\frac{k_x(ms+b)}{m\left(A_cY_0/P_0\right)s^2+\left\{mk_p+b\left(A_cY_0/P_0\right)\right\}s+\left(bk_p+A_c^2)\right)}$

c) $\dot{y}_{ss}=\frac{A_c k_x}{bk_p+A_c^2}$ (for both parts (a) and (b)); $p_{ss}=\frac{bk_x}{bk_p+A_c^2}$

2. For the hydraulic valve and cylinder system shown, the cylinder extends $(y(t)>0)$ as the valve spool is moved to the right $(x(t)>0)$. During this process, the valve-spool displacement is related to the cylinder displacement $(y(t)\sim \text{in.})$ and cap-end pressure $(p(t)\sim \text{psi})$ by the differential equations shown.

$$\ddot{y}+40\dot{y}-p=0$$
$$\dot{y}+\dot{p}+\tfrac{1}{2}p=100x(t)$$

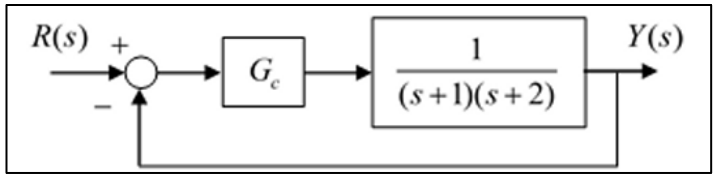
a) Find the cylinder displacement and pressure transfer functions $\frac{Y}{X}(s)$ and $\frac{P}{X}(s)$. b) Find the steady-state speed and pressure associated with a unit step valve-spool input $\left(X(s)=\frac{1}{s}\right)$.

Answers:

a) $\frac{Y}{X}(s)=\frac{100}{s\left(s^2+40.5s+21\right)}$, $\frac{P}{X}(s)=\frac{100\left(s+40\right)}{s^2+40.5s+21}$;

b) $\dot{y}_{ss}=\frac{100}{21}\approx 4.76$ (in/sec), $p_{ss}=\frac{100(40)}{21}\approx 190$ (psi)

3. The block diagram shows a second-order system with a compensator $G_c(s)$. Sketch the root locus diagrams of the closed loop system for the following proportional-integral (PI) and phase-lead (PL) compensator models.



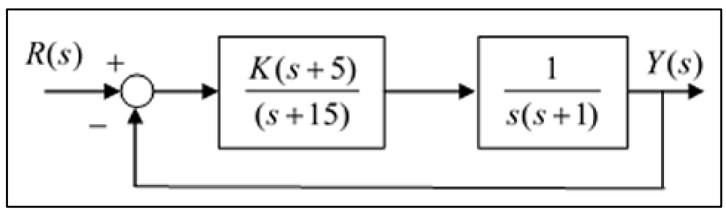
a) $G_c(s)=\dfrac{K(s+3)}{s}$ (proportional-integral); b) $G_c(s)=K\dfrac{(s+3)}{(s+8)}$ (phase-lead)

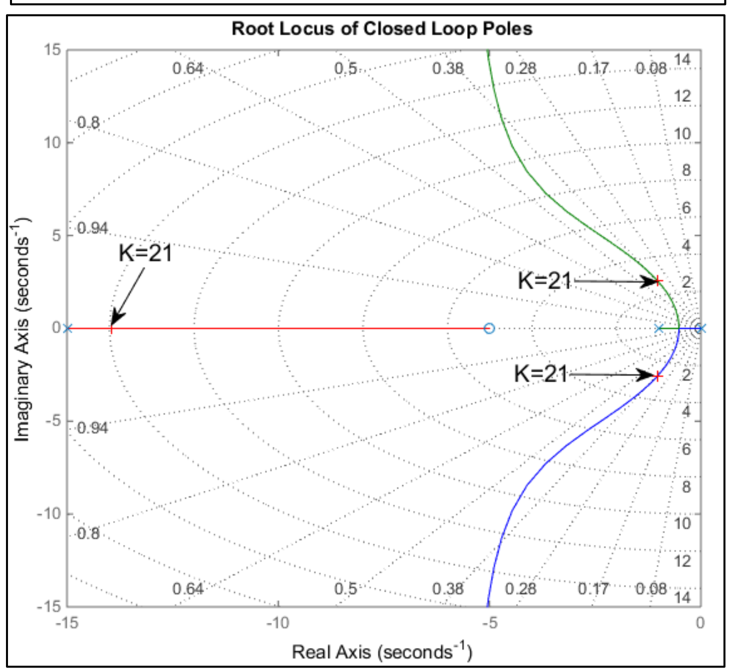
c) Based on your root locus diagrams, which system could be designed to have the smallest settling time? Consider only $K>0$.

Answer:

c) The PI compensator allows the slower closed loop poles to move right (making the system slower) and eventually into the right-half plane (making the system unstable for large enough gains). The PL compensator moves the slower poles farther into the left half plane making the system faster. The PL compensated system is stable for all positive gains. So, the PL compensator can be designed to have the smallest settling time.

4. The block diagram shows a simple motion control system with a phase-lead compensator. The three poles of the closed loop system for $K \approx 21$ are shown on the root locus diagram. For $K \approx 21$ estimate: a) T_s the settling time of the closed loop system, and b) $\%OS$ the percent overshoot of the system to a unit step input.





Pole Locations: -14, $-1.02 \pm 2.55\,i$

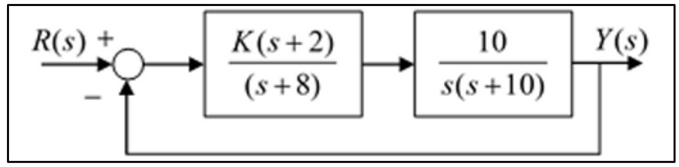
Answers:

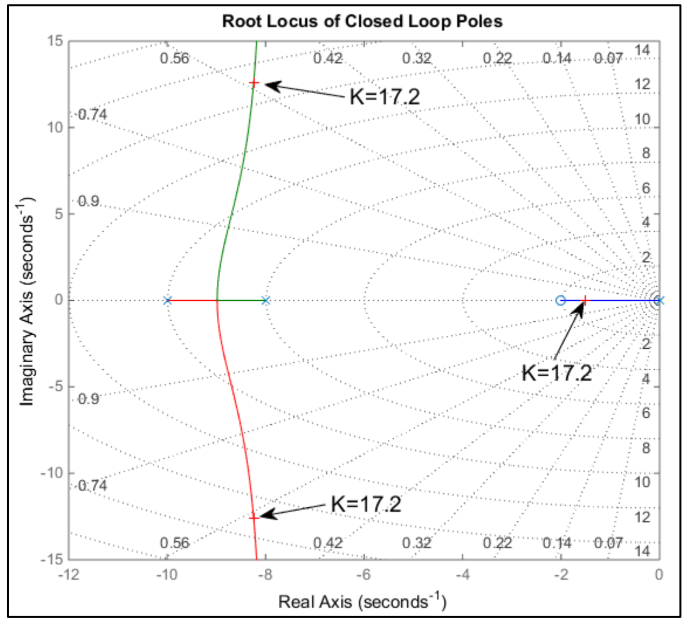
a) $T_s \approx 4$ (sec)

b) 2nd order dominant system with a zero-pole location ratio $\beta = 5$ gives $\%OS \approx 30-32\%$

Note: Simulation shows 33% overshoot and a settling time of 3.84 seconds.

5. The block diagram shows a simple motion control system with a phase-lead (PL) compensator. When $K \approx 17.2$, the root locus diagram of the closed loop system indicates the three roots are located as shown in the diagram. For $K \approx 17.2$, estimate: a) the settling time of all the poles of the closed loop system, b) T_s the settling time of the closed loop system, and c) $\%OS$ the percent overshoot of the closed loop system to a unit step input.





Closed Loop Poles: $-1.51,\ -8.24 \pm 12.6i$

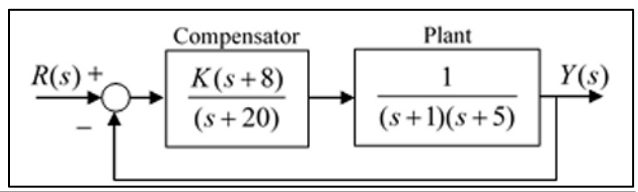
Answers:

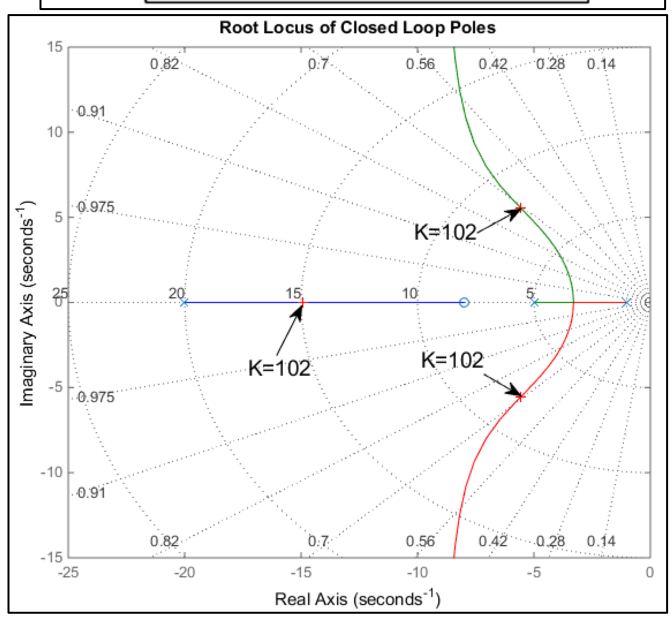
a) real pole: 2.65 (sec), complex poles: 0.485 (sec)

b), c) Difficult to estimate due to near pole-zero cancelation. System could be first-order dominant, second-order, or third order.

Note: Step response shows 3rd order response with suppressed peak response, no overshoot, and settling time of 1.72 (sec).

6. The block diagram shows a simple motion control system with a phase-lead (PL) compensator. When $K \approx 102$, the root locus diagram of the closed loop system indicates the three roots are located as shown in the diagram. For $K \approx 102$ estimate: a) the settling time of each of the closed loop poles, b) T_s the settling time of the closed loop system, c) $\%OS$ the percent overshoot of the system to a unit step input. d) How will the root locus diagram change if the transfer function of the compensator is changed to $G_c(s) = K(s+16)/(s+20)$?





Closed Loop Poles: $-5.56 \pm 5.54i,\ -14.9$

Answers:

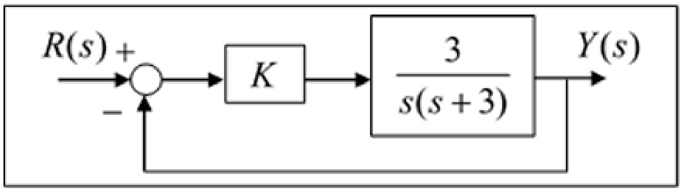
a) 0.72 (sec), 0.27 (sec); b) 0.72 (sec)

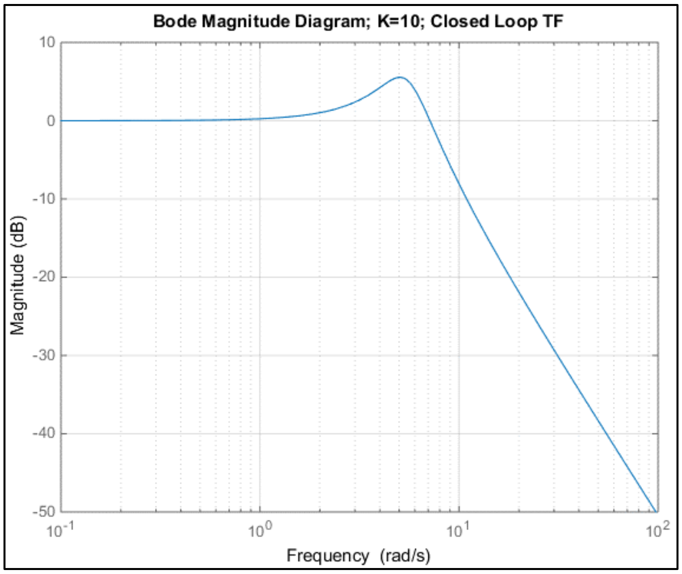
c) 10% (2nd order dominant with $\zeta \approx 0.7$, $\beta \approx 1.44$)

d) moves zero away from the complex poles, but the asymptote of the complex poles moves right.

Note: Simulation shows 8.3% overshoot and a settling time of 0.725 seconds.

7. The block diagram represents a simple motion control system with proportional control. a) Sketch the root locus diagram of the closed loop system. b) Given the Bode magnitude diagram of the closed loop transfer function for $K = 10$, estimate the bandwidth of the system for this value of K. c) How will the magnitude at resonance and the bandwidth of the closed loop system change as K is increased beyond 10?



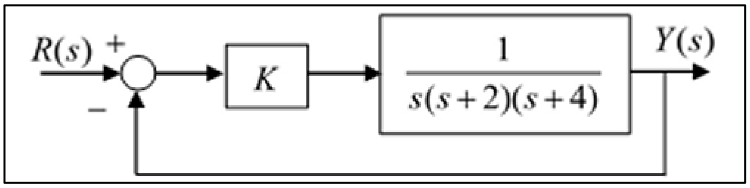


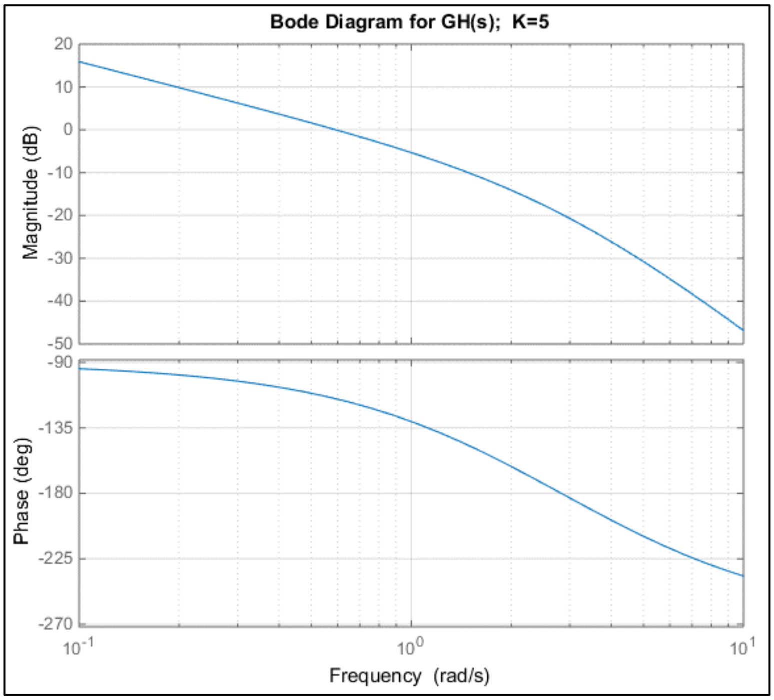
Answers:

b) $BW \approx 8$ (rad/s)

c) Magnitude at resonance and bandwidth will increase as the value of K is increased. Complex poles move toward increased frequency and decreased damping ratio.

8. The block diagram shows a simple motion control system with proportional control. Using the Bode diagram of $GH(s)$ for $K = 5$, estimate a) GM and PM the gain and phase margins of the closed-loop system for this value of K, and b) the range of the gain K required to make the closed-loop system ***unstable***.



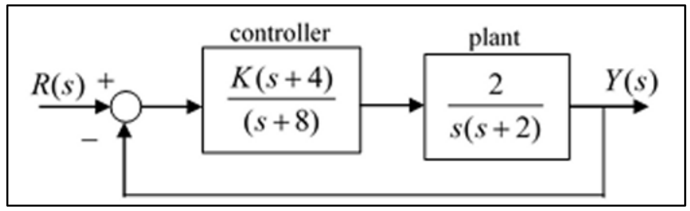


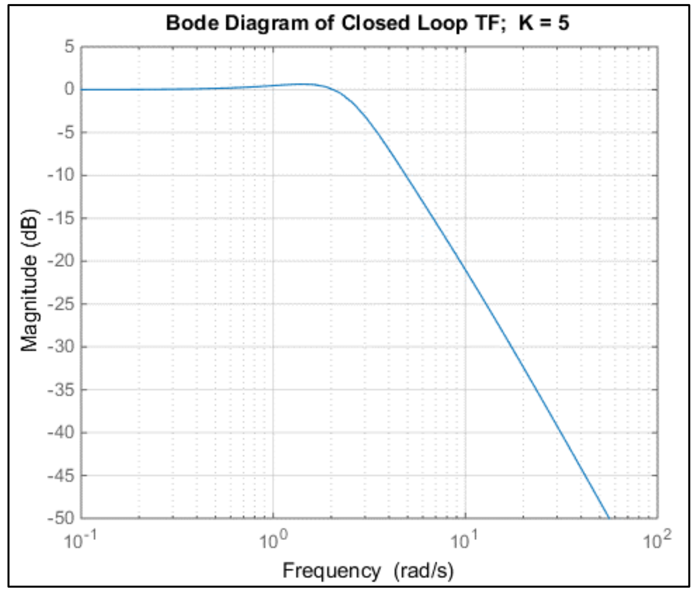
Answers:

a) $GM \approx 20$ (dB); $PM \approx 65$ (deg) (system is stable)

b) $K > 50$ makes the closed loop system unstable.

9. The block diagram shows a simple motion control system with a phase-lead (PL) controller. a) Sketch the root locus diagram of the closed loop system. b) Using the Bode magnitude diagram of the closed loop transfer function for $K = 5$, estimate the bandwidth (BW) of the system for this value of K. c) How will the magnitude at resonance (M_r) and the bandwidth (BW) of the closed loop system change as the value of K is increased beyond 5.



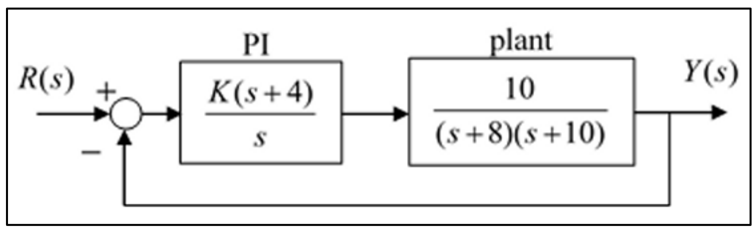


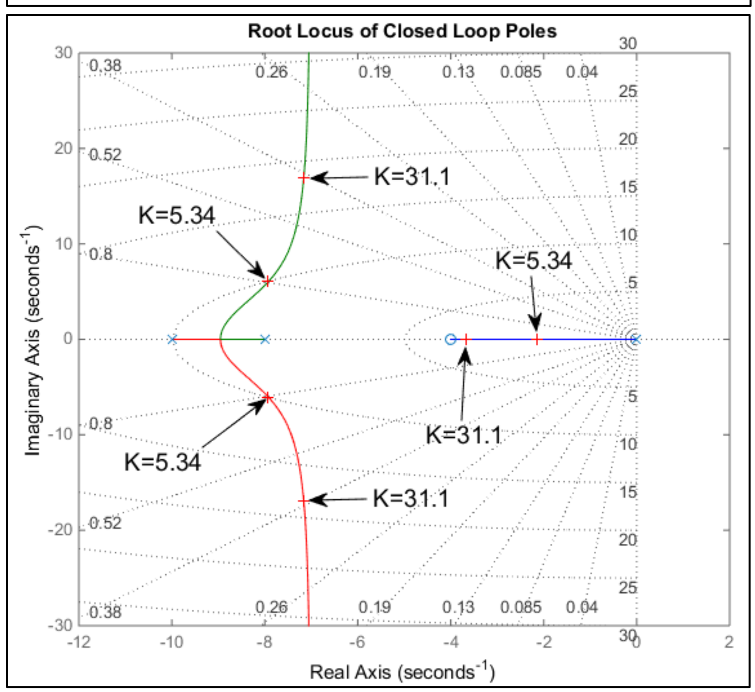
Answers:

b) $BW \approx 3$ (rad/s)

c) Magnitude at resonance and bandwidth will increase as the value of K is increased. Complex poles move toward increased frequency and decreased damping ratio.

10. The block diagram shows a motion control system with a PI compensator. The root locus diagram shows poles of the closed loop system for two different K values, $K \approx 5.34$ and $K \approx 31.1$. a) Estimate the settling time of ***all*** the poles of the closed loop system for $K \approx 5.34$. Then estimate the settling time of the closed loop system for $K \approx 5.34$. b) Estimate the percent overshoot of the closed loop system to a unit step input when $K \approx 31.1$. c) How will the magnitude at resonance (M_r) and the bandwidth (BW) of the closed loop system change as the value of K is increased beyond $K \approx 31.1$? d) How will the root locus diagram change if the following PID compensator is used? $G_c(s) = \dfrac{K(s+4)(s+12)}{s}$





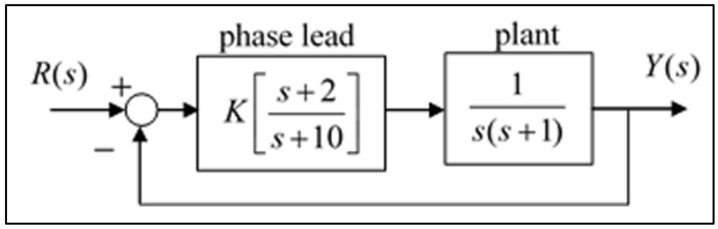
Closed Loop Poles: $K \approx 5.34$: $\{-2.15, -7.92 \pm 6.04i\}$ and $K \approx 31.1$: $\{-3.68, -7.16 \pm 16.9i\}$

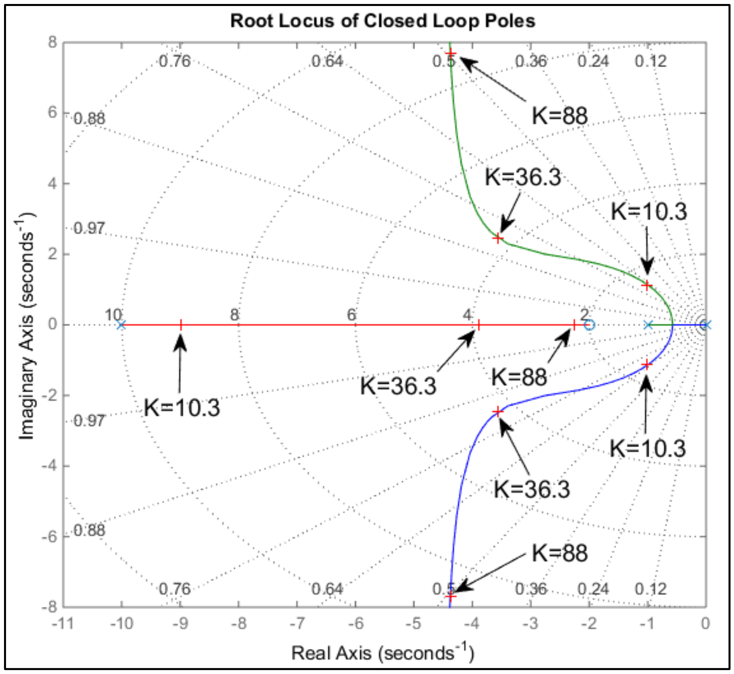
Answers:

a) 1.9 (sec), 0.51 (sec) and 1.9 (sec)
b) If significant near pole-zero cancelation: 28%. Otherwise, 0%.
c) M_r and BW will continue to increase as the near pole-zero cancelation gets more significant and as the complex poles continue to move towards less damping and higher frequency.
d) The two branches that move to infinity with the PI compensator will break into the real axis to the left of $s=-12$ with the PID compensator. The third branch remains the same.

Note: simulation shows a) no overshoot with settling time of 1.62 (sec); b) 19.5% overshoot.

11. The block diagram shows a motion control system with a phase-lead compensator. The root locus diagram shows poles of the closed loop system for $K \approx \{10.3, 36.3, 88\}$. a) Estimate the settling time and percent overshoot for $K \approx 10.3$. b) Estimate the settling time and percent overshoot for $K \approx 88$. c) Why is it difficult to estimate the percent overshoot (without software assistance) for $K \approx 36.3$? d) How will the magnitude at resonance (M_r) and the bandwidth (BW) of the closed loop system change as the value of K is increased beyond $K = 89$?





Closed loop poles:

$K \approx 10.3$: $\{-1.0 \pm 1.14i, -9.0\}$

$K \approx 36.3$: $\{-3.55 \pm 2.46i, -3.89\}$

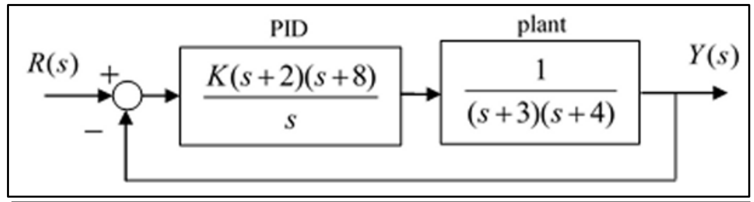
$K \approx 88$: $\{-2.25, -4.38 \pm 7.69i\}$

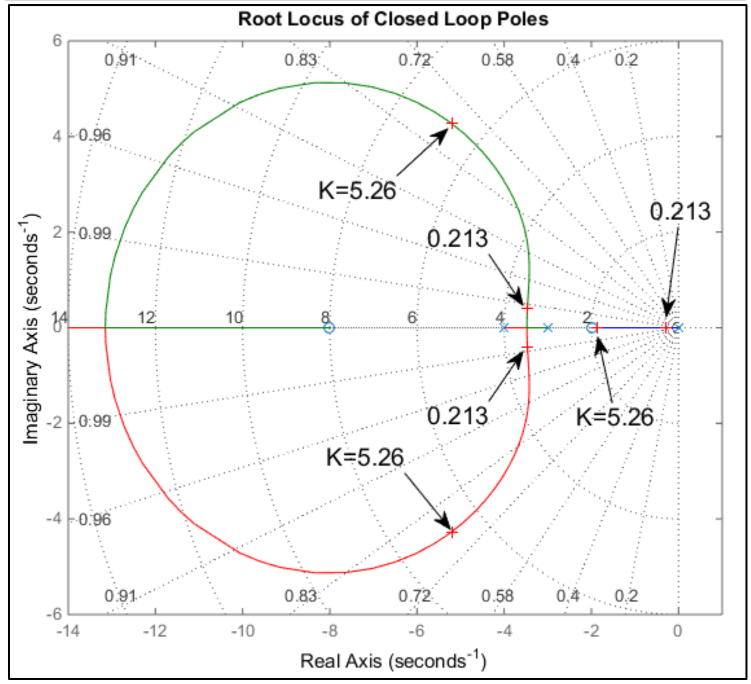
Answers:

a) second order dominant with $\zeta \approx 0.64$, $a = 2$ and $\zeta\omega_n = 1 \Rightarrow$ 4 (sec), $10-12\%$
b) near pole-zero cancelation, second order system with $\zeta \approx 0.5$, $\zeta\omega_n = 4.38 \Rightarrow$ 0.91 (sec), $16-17\%$
c) three poles clustered together, so system has 3rd order response
d) M_r and BW will continue to increase as the near pole-zero cancelation becomes more obvious and the complex poles move towards higher frequencies and lower damping ratios.

Note: simulation shows a) settling time of 3.56 (sec) and 9.85% overshoot, b) settling time of 0.7 (sec) and 25.4% overshoot.

12. The block diagram shows a motion control system with a PID compensator. The root locus diagram shows poles of the closed loop system for $K = 0.213$ and $K = 5.26$. a) For $K = 0.213$, estimate the settling time and the percent overshoot to a unit step input. b) For $K = 5.26$, estimate the settling time and the percent overshoot to a unit step input. c) How will the bandwidth (BW) of the closed loop system change as the value of K is increased beyond $K = 5.26$?





Closed loop poles:

$K = 0.213$: $\{-0.279, -3.47 \pm 0.416\}$

$K = 5.26$: $\{-1.86, -5.20 \pm 4.27\}$

Answers:

a) 1st order dominance: $\%OS = 0\%$ and $T_s = 14.3$ (sec)

b) near pole-zero cancelation, second order system $\zeta \approx 0.77$: $\%OS \approx 3-4\%$, $T_s = 0.77$ (sec)

c) Once the first near pole-zero cancellation takes effect, BW will be determined by the other two poles which increase in frequency until the break point at $s \approx -13$. At this point one of the poles decreases in frequency and the other increases. The pole that decreases ultimately causes a second near pole-zero cancellation. So, at large enough K values the system is first order dominant with an ever-increasing BW.

Note: simulation shows a) $\%OS = 0\%$ and $T_s = 13.9$ (sec), b) $\%OS \approx 1\%$ and $T_s = 0.387$ (sec)