

Example #2a – Intermediate Dynamics: Velocity

Reference frames:

$$R: \underline{i}, \underline{j}, \underline{k} \text{ (fixed frame)}$$

$$F: \underline{e}_1, \underline{e}_2, \underline{k} \text{ (fixed in the rotating frame)}$$

Find:

${}^R \underline{v}_P$... the **velocity** of point P in R using **direct differentiation**

Solution:

To find the velocity of P , differentiate the position vector of P . Here, \underline{e}_r is a unit vector pointing from the center of the disk towards P .

$$\begin{aligned} {}^R \underline{v}_P &= \frac{{}^R d}{dt}(\underline{r}_{P/O}) = \frac{{}^R d}{dt}(\ell \underline{e}_2 + a \underline{e}_r) = \ell \frac{{}^R d}{dt}(\underline{e}_2) + a \frac{{}^R d}{dt}(\underline{e}_r) \quad \text{(expressed in mixed frames)} \\ &= \ell(\Omega \underline{k} \times \underline{e}_2) + a[(\omega \underline{e}_2 + \Omega \underline{k}) \times \underline{e}_r] = -\ell \Omega \underline{e}_1 + a[(\omega \underline{e}_2 + \Omega \underline{k}) \times (-C_\theta \underline{e}_1 + S_\theta \underline{k})] \\ &= -\ell \Omega \underline{e}_1 + a[\omega C_\theta \underline{k} + S_\theta \omega \underline{e}_1 - \Omega C_\theta \underline{e}_2] \end{aligned}$$

So,

$${}^R \underline{v}_P = (a\omega S_\theta - \ell\Omega) \underline{e}_1 - (a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k}$$

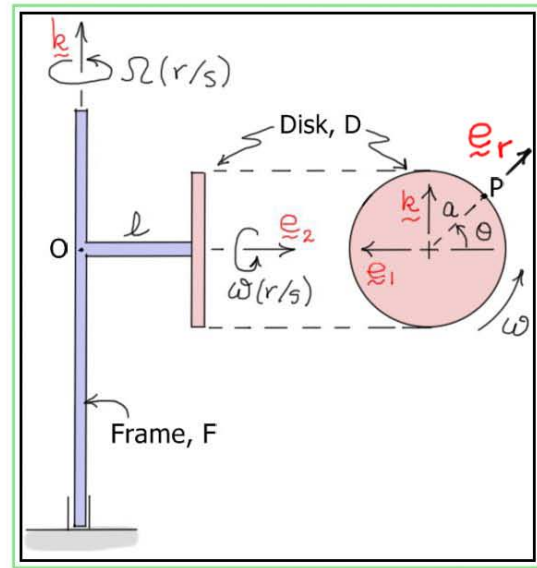
(expressed in frame F)

Or,

$$\begin{aligned} {}^R \underline{v}_P &= \frac{{}^R d}{dt}(\underline{r}_{P/O}) = \frac{{}^R d}{dt}(-aC_\theta \underline{e}_1 + \ell \underline{e}_2 + aS_\theta \underline{k}) \\ &= (aS_\theta \omega) \underline{e}_1 - aC_\theta \frac{{}^R d}{dt}(\underline{e}_1) + \ell \frac{{}^R d}{dt}(\underline{e}_2) + (aS_\theta \omega) \underline{k} + \underbrace{(aS_\theta)}_{\text{zero}} \frac{{}^R d}{dt}(\underline{k}) \\ &= (aS_\theta \omega) \underline{e}_1 - aC_\theta (\Omega \underline{k} \times \underline{e}_1) + \ell (\Omega \underline{k} \times \underline{e}_2) + (aS_\theta \omega) \underline{k} \\ &= (aS_\theta \omega) \underline{e}_1 - aC_\theta \Omega \underline{e}_2 - \ell \Omega \underline{e}_1 + (aS_\theta \omega) \underline{k} \end{aligned}$$

So,

$${}^R \underline{v}_P = (a\omega S_\theta - \ell\Omega) \underline{e}_1 - (a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k}$$



Aside:

$$\begin{aligned} \frac{d}{dt}(-aC_\theta) &= -a(-S_\theta \dot{\theta}) = a\omega S_\theta \\ \frac{d}{dt}(aS_\theta) &= a(C_\theta \dot{\theta}) = a\omega C_\theta \end{aligned}$$