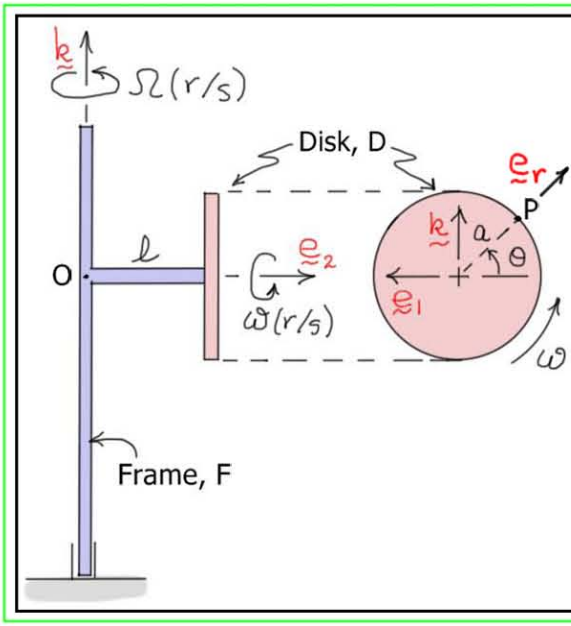
Example #2b – Intermediate Dynamics: Velocity

Reference frames:

$R: \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$F: \underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{k}$ (fixed in the rotating frame)



Find:

${}^R\underset{\sim}{v}_P$ … the ***velocity*** of point P in R using ***direct differentiation***

Solution:

To find the velocity of P, differentiate the position vector of P. Here, $\underset{\sim}{e}_r$ is a unit vector pointing from the center of the disk towards P. In this solution, we take advantage of the "derivative rule."

$$
{}^R\underset{\sim}{v}_P = \frac{{}^Rd}{dt}\left(\underset{\sim}{r}_{P/O}\right) = \frac{{}^Rd}{dt}\left(\ell\,\underset{\sim}{e}_2 + a\,\underset{\sim}{e}_r\right) = \underbrace{\frac{{}^Dd}{dt}\left(\ell\,\underset{\sim}{e}_2 + a\,\underset{\sim}{e}_r\right)}_{\text{zero, but why?}} + {}^R\underset{\sim}{\omega}_D \times \left(\ell\,\underset{\sim}{e}_2 + a\,\underset{\sim}{e}_r\right)
$$

$$
= \left(\omega\,\underset{\sim}{e}_2 + \Omega\,\underset{\sim}{k}\right) \times \left(\ell\,\underset{\sim}{e}_2 + a\,\underset{\sim}{e}_r\right) = \left(\omega\,\underset{\sim}{e}_2 + \Omega\,\underset{\sim}{k}\right) \times \left(\ell\,\underset{\sim}{e}_2 + a\left(-C_\theta\,\underset{\sim}{e}_1 + S_\theta\,\underset{\sim}{k}\right)\right)
$$

$$
= \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{k} \\ 0 & \omega & \Omega \\ -aC_\theta & \ell & aS_\theta \end{vmatrix} = \left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 + \left(-a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}
$$

So, as before

$$
{}^R\underset{\sim}{v}_P = \left(a\omega S_\theta - \ell\Omega\right)\underset{\sim}{e}_1 - \left(a\Omega C_\theta\right)\underset{\sim}{e}_2 + \left(a\omega C_\theta\right)\underset{\sim}{k}
$$