

Example #3 – Intermediate Dynamics: Acceleration

Reference frames:

$$R: \underline{i}, \underline{j}, \underline{k} \text{ (fixed frame)}$$

$$F: \underline{e}_1, \underline{e}_2, \underline{k} \text{ (fixed in the rotating frame)}$$

Find:

${}^R \underline{a}_P$... the *acceleration* of point P in R using *direct differentiation*

Solution:

To find the acceleration of P , we can differentiate the velocity vector of P . In this solution, we take advantage again of the “derivative rule.”

$${}^R \underline{a}_P = \frac{{}^R d}{dt} ({}^R \underline{v}_P) = \frac{{}^F d}{dt} ({}^R \underline{v}_P) + {}^R \underline{\omega}_F \times ({}^R \underline{v}_P)$$

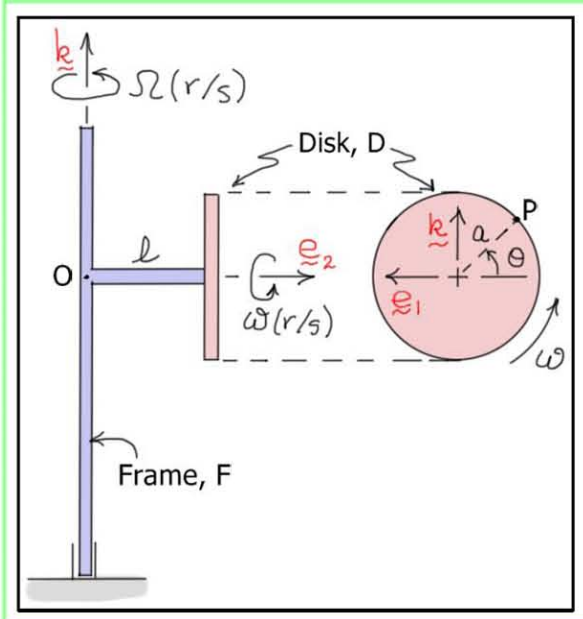
Here,

$$\begin{aligned} \frac{{}^F d}{dt} ({}^R \underline{v}_P) &= \frac{{}^F d}{dt} \left((a\omega S_\theta - \ell\Omega) \underline{e}_1 - (a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k} \right) \\ &= (a\dot{\omega} S_\theta + a\omega^2 C_\theta - \ell\dot{\Omega}) \underline{e}_1 - (a\dot{\Omega} C_\theta - a\Omega\omega S_\theta) \underline{e}_2 \\ &\quad + (a\dot{\omega} C_\theta - a\omega^2 S_\theta) \underline{k} \end{aligned}$$

$$\begin{aligned} {}^R \underline{\omega}_F \times {}^R \underline{v}_P &= (\Omega \underline{k}) \times \left[(a\omega S_\theta - \ell\Omega) \underline{e}_1 - (a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k} \right] \\ &= (a\Omega^2 C_\theta) \underline{e}_1 + ((a\omega S_\theta - \ell\Omega)\Omega) \underline{e}_2 \end{aligned}$$

Substituting these results into the boxed equation gives the result.

$$\begin{aligned} {}^R \underline{a}_P &= \left[a\dot{\omega} S_\theta - \ell\dot{\Omega} + aC_\theta(\omega^2 + \Omega^2) \right] \underline{e}_1 + \left[-a\dot{\Omega} C_\theta + 2a\omega\Omega S_\theta - \ell\Omega^2 \right] \underline{e}_2 \\ &\quad + \left[a\dot{\omega} C_\theta - a\omega^2 S_\theta \right] \underline{k} \end{aligned}$$



Aside:

$$\begin{aligned} \frac{d}{dt} (a\omega S_\theta) &= a\dot{\omega} S_\theta + a\omega (\dot{\theta} C_\theta) \\ &= a\dot{\omega} S_\theta + a\omega^2 C_\theta \\ \frac{d}{dt} (a\Omega S_\theta) &= a\dot{\Omega} S_\theta + a\Omega\omega C_\theta \end{aligned}$$