

## Example #4 – Intermediate Dynamics: Velocity – Two-Point Formula

Reference frames:

$R: \underline{i}, \underline{j}, \underline{k}$  (fixed frame)

$F: \underline{e}_1, \underline{e}_2, \underline{k}$  (rotating frame)

Find:

${}^R \underline{v}_P$  ... the **velocity** of point  $P$  in  $R$  using the **two-point formula**

Solution:

To find the velocity of  $P$ , we can use the formula that relates the velocity of two points on a rigid body.

Result from a previous example:  ${}^R \underline{\omega}_D = \omega \underline{e}_2 + \Omega \underline{k}$

Using the formula that relates the velocity of two points of a rigid body, write

$${}^R \underline{v}_P = {}^R \underline{v}_Q + {}^R \underline{v}_{P/Q} \quad (\text{here, } Q \text{ represents the center of the disk})$$

(for two points fixed on a rigid body)

where,

$${}^R \underline{v}_Q = -\ell \Omega \underline{e}_1 \quad (Q \text{ has circular motion around } O)$$

$${}^R \underline{v}_{P/Q} = {}^R \underline{\omega}_D \times \underline{r}_{P/Q} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{k} \\ 0 & \omega & \Omega \\ -aC_\theta & 0 & aS_\theta \end{vmatrix}$$

$$= (a\omega S_\theta) \underline{e}_1 + (-a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k}$$

Adding these two results gives

$${}^R \underline{v}_P = (a\omega S_\theta - \ell\Omega) \underline{e}_1 - (a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k}$$

