

Example #5 – Intermediate Dynamics: Acceleration – Two-Point Formula

Reference frames:

$R: \underline{i}, \underline{j}, \underline{k}$ (fixed frame)

$F: \underline{e}_1, \underline{e}_2, \underline{k}$ (rotating frame)

Find:

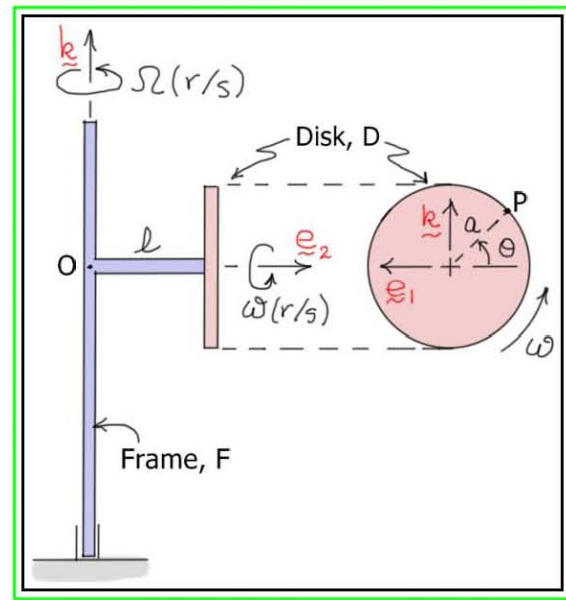
${}^R \underline{a}_P$... the **acceleration** of point P in R using the **two-point formula**

Solution:

To find the acceleration of P , use the formula that relates the accelerations of two points of a rigid body.

$${}^R \underline{\omega}_D = \omega \underline{e}_2 + \Omega \underline{k} \quad {}^R \underline{\alpha}_D = -\omega \Omega \underline{e}_1 + \dot{\omega} \underline{e}_2 + \dot{\Omega} \underline{k}$$

$${}^R \underline{v}_{P/Q} = (a\omega S_\theta) \underline{e}_1 + (-a\Omega C_\theta) \underline{e}_2 + (a\omega C_\theta) \underline{k}$$



Using the two-point formula, write:

$${}^R \underline{a}_P = {}^R \underline{a}_Q + ({}^R \underline{\alpha}_D \times \underline{r}_{P/Q}) + ({}^R \underline{\omega}_D \times {}^R \underline{v}_{P/Q})$$

Here,

(for two points fixed on a rigid body)

$${}^R \underline{a}_Q = -l\dot{\Omega} \underline{e}_1 - l\Omega^2 \underline{e}_2 \quad (Q \text{ has circular motion around } O)$$

$${}^R \underline{\alpha}_D \times \underline{r}_{P/Q} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{k} \\ -\omega \Omega & \dot{\omega} & \dot{\Omega} \\ -aC_\theta & 0 & aS_\theta \end{vmatrix} = (a\dot{\omega} S_\theta) \underline{e}_1 + (a\omega \Omega S_\theta - a\dot{\Omega} C_\theta) \underline{e}_2 + (a\dot{\omega} C_\theta) \underline{k}$$

$${}^R \underline{\omega}_D \times {}^R \underline{v}_{P/Q} = \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{k} \\ 0 & \omega & \Omega \\ a\omega S_\theta & -a\Omega C_\theta & a\omega C_\theta \end{vmatrix} = (a\omega^2 C_\theta + a\Omega^2 C_\theta) \underline{e}_1 + (a\omega \Omega S_\theta) \underline{e}_2 + (-a\omega^2 S_\theta) \underline{k}$$

Substituting these results into the boxed equation gives

$${}^R \underline{a}_P = \left[a\dot{\omega} S_\theta - l\dot{\Omega} + aC_\theta (\omega^2 + \Omega^2) \right] \underline{e}_1 + \left[-a\dot{\Omega} C_\theta + 2a\omega \Omega S_\theta - l\Omega^2 \right] \underline{e}_2 + \left[a\dot{\omega} C_\theta - a\omega^2 S_\theta \right] \underline{k}$$