

Example #7 – Intermediate Dynamics: Points Moving on Bodies

Reference frames:

$$R: \underline{\hat{i}}, \underline{\hat{j}}, \underline{\hat{k}} \text{ (fixed frame)}$$

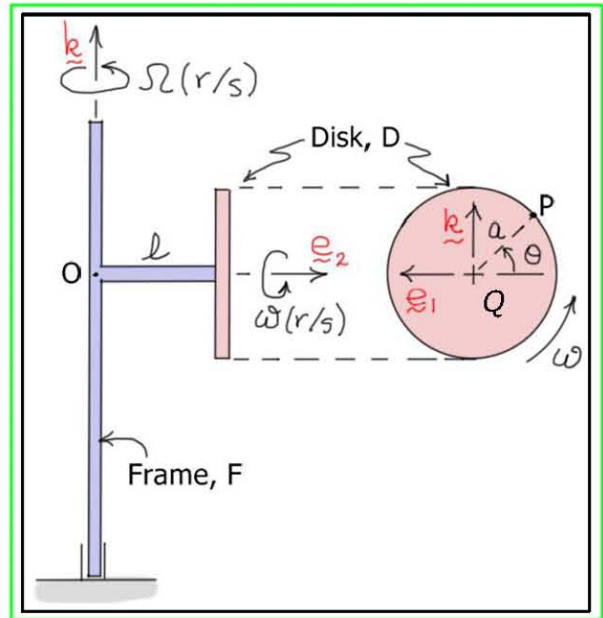
$$F: \underline{\hat{e}}_1, \underline{\hat{e}}_2, \underline{\hat{k}} \text{ (rotating frame)}$$

Find:

$${}^R \underline{v}_P \dots \text{ the velocity of point } P \text{ in } R \text{ using the formula for points moving on bodies}$$

Solution:

To find the velocity of P , use the formula for points moving on a rigid body. In this example, think of point P as moving on the frame F (rather than as a point fixed on the disk).



$${}^R \underline{v}_P = {}^R \underline{v}_{\hat{P}} + {}^F \underline{v}_P \text{ (here, } \hat{P} \text{ is fixed on } F \text{ and coincides with } P \text{)}$$

Here,

$${}^R \underline{v}_{\hat{P}} = \underbrace{{}^R \underline{v}_O}_{\text{zero}} + {}^R \underline{v}_{\hat{P}/O} = {}^R \underline{\omega}_F \times \underline{r}_{\hat{P}/O} = \Omega \underline{\hat{k}} \times (-a C_\theta \underline{\hat{e}}_1 + l \underline{\hat{e}}_2 + a S_\theta \underline{\hat{k}})$$

$${}^R \underline{v}_{\hat{P}} = \Omega (-a C_\theta \underline{\hat{e}}_2 - l \underline{\hat{e}}_1)$$

$${}^F \underline{v}_P = \underbrace{{}^F \underline{v}_Q}_{\text{zero}} + {}^F \underline{v}_{P/Q} = {}^F \underline{\omega}_D \times \underline{r}_{P/Q} = \omega \underline{\hat{e}}_2 \times (-a C_\theta \underline{\hat{e}}_1 + a S_\theta \underline{\hat{k}})$$

$${}^F \underline{v}_P = \omega (a C_\theta \underline{\hat{k}} + a S_\theta \underline{\hat{e}}_1)$$

Adding these two results gives

$${}^R \underline{v}_P = (a \omega S_\theta - l \Omega) \underline{\hat{e}}_1 + (-a \Omega C_\theta) \underline{\hat{e}}_2 + (a \omega C_\theta) \underline{\hat{k}}$$