

Example #8 – Intermediate Dynamics: Point Moving on a Body

Reference frames:

$R: \underline{i}, \underline{j}, \underline{k}$ (fixed frame)

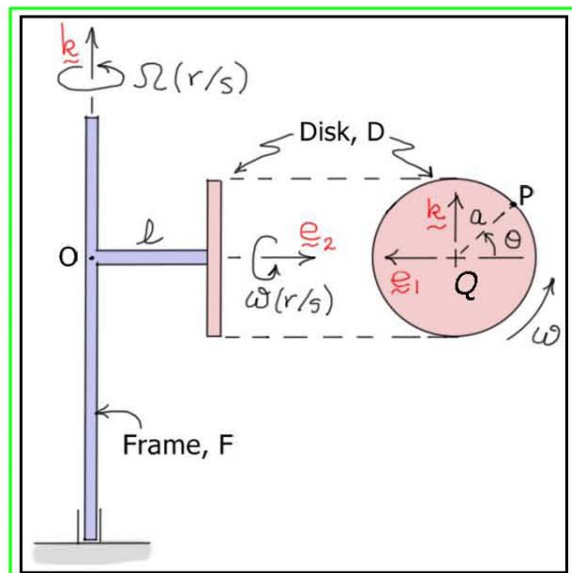
$F: \underline{e}_1, \underline{e}_2, \underline{k}$ (rotating frame)

Find:

${}^R \underline{a}_P$... the **acceleration** of point P in R using the **formula for points moving on bodies**

Solution:

To find the **acceleration** of P , use the formula for a **point moving on a rigid body**. In this example, think of point P as **moving on the frame F** (rather than as a point fixed on the disk).



$${}^R \underline{a}_P = {}^R \underline{a}_{\hat{P}} + {}^F \underline{a}_P + 2({}^R \underline{\omega}_F \times {}^F \underline{v}_P) \quad (\text{here, } \hat{P} \text{ is fixed on } F \text{ and coincides with } P)$$

where,

$${}^R \underline{a}_{\hat{P}} = \underbrace{{}^R \underline{a}_O}_{\text{zero}} + {}^R \underline{a}_{\hat{P}/O} = [{}^R \underline{\alpha}_F \times \underline{r}_{\hat{P}/O}] + [{}^R \underline{\omega}_F \times \underline{r}_{\hat{P}/O}] \quad (\hat{P} \text{ and } O \text{ are both fixed on } F)$$

$$= [\dot{\Omega} \underline{k} \times (-a C_\theta \underline{e}_1 + l \underline{e}_2 + a S_\theta \underline{k})] + \left[\Omega \underline{k} \times \underbrace{\Omega (-a C_\theta \underline{e}_2 - l \underline{e}_1)}_{\text{from previous example}} \right]$$

$$= \dot{\Omega} (-a C_\theta \underline{e}_2 - l \underline{e}_1) + \Omega^2 (a C_\theta \underline{e}_1 - l \underline{e}_2)$$

$${}^R \underline{a}_{\hat{P}} = (a \Omega^2 C_\theta - l \dot{\Omega}) \underline{e}_1 - (l \Omega^2 + a \dot{\Omega} C_\theta) \underline{e}_2$$

$${}^F \underline{a}_P = [{}^F \underline{\alpha}_D \times \underline{r}_{P/Q}] + [{}^F \underline{\omega}_D \times \underline{r}_{P/Q}] \quad (\text{P and Q are fixed on the disk})$$

$$= [\dot{\omega} \underline{e}_2 \times (-a C_\theta \underline{e}_1 + a S_\theta \underline{k})] + [\omega \underline{e}_2 \times (\omega (a C_\theta \underline{k} + a S_\theta \underline{e}_1))]]$$

$$= \dot{\omega} (a C_\theta \underline{k} + a S_\theta \underline{e}_1) + \omega^2 (a C_\theta \underline{e}_1 - a S_\theta \underline{k})$$

$${}^F \underline{a}_P = (a \dot{\omega} S_\theta + a \omega^2 C_\theta) \underline{e}_1 + (a \dot{\omega} C_\theta - a \omega^2 S_\theta) \underline{k}$$

$$2({}^R \underline{\omega}_F \times {}^F \underline{v}_P) = 2 \Omega \underline{k} \times \omega (a C_\theta \underline{k} + a S_\theta \underline{e}_1) = 2 a \omega \Omega S_\theta \underline{e}_2$$

Adding these three results gives

$${}^R \underline{a}_P = (a \dot{\omega} S_\theta - l \dot{\Omega} + a C_\theta (\omega^2 + \Omega^2)) \underline{e}_1 + (2 a \omega \Omega S_\theta - a \dot{\Omega} C_\theta - l \Omega^2) \underline{e}_2 + (a \dot{\omega} C_\theta - a \omega^2 S_\theta) \underline{k}$$