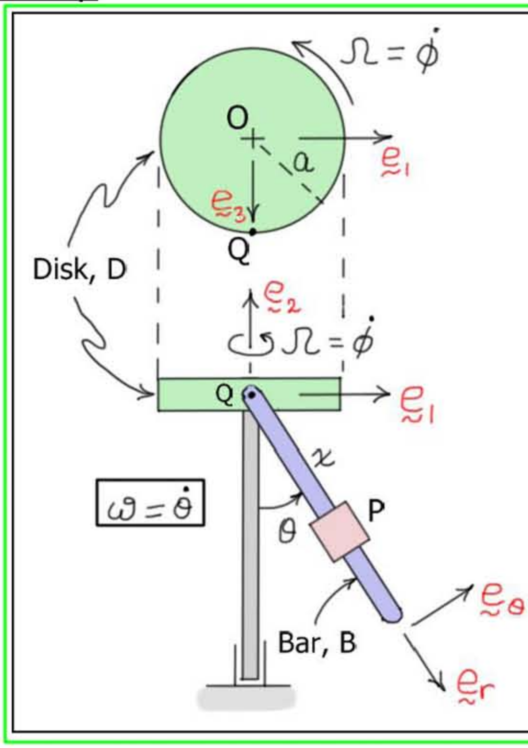
Example #9 – Intermediate Dynamics: Point Moving on a Body

Reference frames: (R is a fixed frame)

D: $\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3$ (rotating with disk D)

B: $\underset{\sim}{e}_r, \underset{\sim}{e}_\theta, \underset{\sim}{e}_3$ (rotating with the bar B)



Find:

${}^R\underset{\sim}{v}_P$ … the ***velocity*** of point P in R.

Solution:

To find the velocity of P, use the formula for a point moving on a rigid body. ($\hat{P}$ is fixed on B and coincides with P)

$$\boxed{{}^R\underset{\sim}{v}_P = {}^R\underset{\sim}{v}_{\hat{P}} + {}^B\underset{\sim}{v}_P}$$

Here,

$${}^R\underset{\sim}{v}_{\hat{P}} = {}^R\underset{\sim}{v}_Q + {}^R\underset{\sim}{v}_{\hat{P}/Q} = \left({}^R\underset{\sim}{\omega}_D \times \underset{\sim}{r}_{Q/O}\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{r}_{\hat{P}/Q}\right)$$

(Q and O are both fixed on D)

(P_hat and Q are both fixed on B)

$$= \left(\Omega\, \underset{\sim}{e}_2 \times a\, \underset{\sim}{e}_3\right) + \begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{e}_2 & \underset{\sim}{e}_3 \\ 0 & \Omega & \omega \\ xS_\theta & -xC_\theta & 0 \end{vmatrix}$$

$$\boxed{{}^R\underset{\sim}{v}_{\hat{P}} = \left(a\Omega + x\omega C_\theta\right)\underset{\sim}{e}_1 + \left(x\omega S_\theta\right)\underset{\sim}{e}_2 + \left(-x\Omega S_\theta\right)\underset{\sim}{e}_3}$$

$$\boxed{{}^B\underset{\sim}{v}_P = \underbrace{{}^B\underset{\sim}{v}_Q}_{\text{zero}} + {}^B\underset{\sim}{v}_{P/Q} = \dot{x}\,\underset{\sim}{e}_r = \dot{x}\left(S_\theta\, \underset{\sim}{e}_1 - C_\theta\, \underset{\sim}{e}_2\right)}$$

Adding these two results gives

$$\boxed{{}^R\underset{\sim}{v}_P = \left(a\Omega + x\omega C_\theta + \dot{x}S_\theta\right)\underset{\sim}{e}_1 + \left(x\omega S_\theta - \dot{x}C_\theta\right)\underset{\sim}{e}_2 - \left(x\Omega S_\theta\right)\underset{\sim}{e}_3}$$