

Example #10 – Intermediate Dynamics: Point Moving on a Body

Reference frames: (R is a fixed frame)

D : $\underline{e}_1, \underline{e}_2, \underline{e}_3$ (rotating with disk D)

B : $\underline{e}_r, \underline{e}_\theta, \underline{e}_3$ (rotating with the bar B)

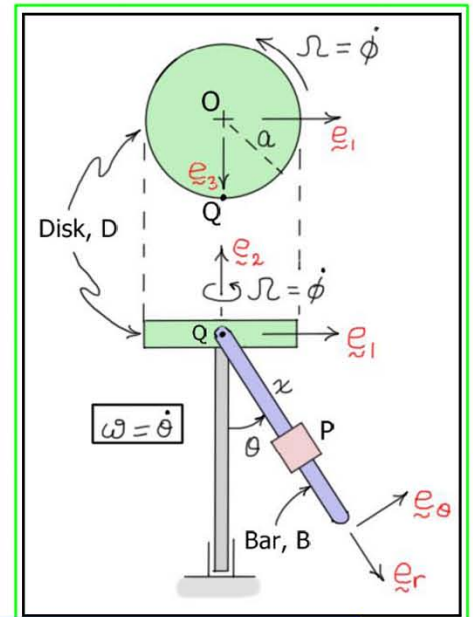
Find:

${}^R \underline{a}_P$... the **acceleration** of point P in R .

Solution:

To find the **acceleration** of P , use the formula for a point moving on a rigid body. First need to calculate the angular acceleration of the bar.

Angular Acceleration of Bar, B



$${}^R \underline{\alpha}_B = \frac{{}^R d({}^R \underline{\omega}_B)}{dt} = \frac{{}^R d(\Omega \underline{e}_2 + \omega \underline{e}_3)}{dt} = \dot{\Omega} \underline{e}_2 + \dot{\omega} \underline{e}_3 + \omega (\Omega \underline{e}_2 \times \underline{e}_3) = \omega \Omega \underline{e}_1 + \dot{\Omega} \underline{e}_2 + \dot{\omega} \underline{e}_3$$

Now write

$${}^R \underline{a}_P = {}^R \underline{a}_{\hat{P}} + {}^B \underline{a}_P + 2({}^R \underline{\omega}_B \times {}^B \underline{v}_P) \quad (\hat{P} \text{ is fixed on } B \text{ and coincides with } P)$$

Here,

$${}^R \underline{a}_{\hat{P}} = {}^R \underline{a}_Q + {}^R \underline{a}_{\hat{P}/Q} = (a\dot{\Omega} \underline{e}_1 - a\Omega^2 \underline{e}_3) + ({}^R \underline{\alpha}_B \times \underline{r}_{\hat{P}/Q}) + ({}^R \underline{\omega}_B \times \underline{v}_{\hat{P}/Q}) \quad (P_{\text{hat}} \text{ and } Q \text{ are fixed on } B)$$

$$= (a\dot{\Omega} \underline{e}_1 - a\Omega^2 \underline{e}_3) + \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ \omega \Omega & \dot{\Omega} & \dot{\omega} \\ x S_\theta & -x C_\theta & 0 \end{vmatrix} + \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ 0 & \Omega & \omega \\ x \omega C_\theta & x \omega S_\theta & -x \Omega S_\theta \end{vmatrix}$$

$${}^R \underline{a}_{\hat{P}} = (a\dot{\Omega} + x\dot{\omega}C_\theta - xS_\theta(\Omega^2 + \omega^2)) \underline{e}_1 + (x\dot{\omega}S_\theta + x\omega^2C_\theta) \underline{e}_2 - (a\Omega^2 + 2x\omega\Omega C_\theta + x\dot{\Omega}S_\theta) \underline{e}_3$$

$${}^B \underline{a}_P = \underbrace{{}^B \underline{a}_Q}_{\text{zero}} + {}^B \underline{a}_{P/Q} = \ddot{x} \underline{e}_r = \ddot{x} (S_\theta \underline{e}_1 - C_\theta \underline{e}_2)$$

$$2({}^R \underline{\omega}_B \times {}^B \underline{v}_P) = 2 \begin{vmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \\ 0 & \Omega & \omega \\ \dot{x} S_\theta & -\dot{x} C_\theta & 0 \end{vmatrix} = 2 [(\dot{x}\omega C_\theta) \underline{e}_1 + (\dot{x}\omega S_\theta) \underline{e}_2 - (\dot{x}\Omega S_\theta) \underline{e}_3]$$

Adding these three results gives

$${}^R \underline{a}_P = (a\dot{\Omega} + x\dot{\omega}C_\theta - xS_\theta(\Omega^2 + \omega^2) + \ddot{x}S_\theta + 2\dot{x}\omega C_\theta) \underline{e}_1 + (x\dot{\omega}S_\theta + x\omega^2C_\theta - \ddot{x}C_\theta + 2\dot{x}\omega S_\theta) \underline{e}_2 - (a\Omega^2 + 2x\omega\Omega C_\theta + x\dot{\Omega}S_\theta + 2\dot{x}\Omega S_\theta) \underline{e}_3$$