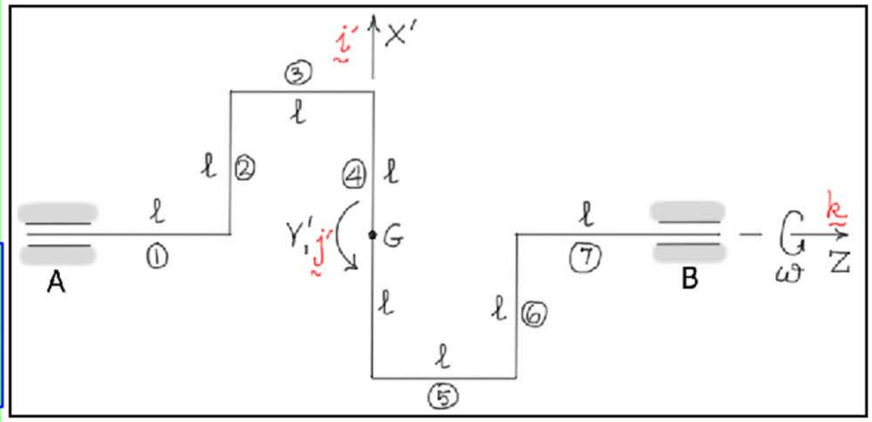
Example #14 – Intermediate Dynamics: Simple Crank Shaft – Bearing Loads

The figure shows a ***simple crank shaft*** consisting of ***seven segments***, each considered to be a ***slender bar***. Each segment of ***length*** ℓ has ***mass*** m. There are six segments of length ℓ and one segment of length 2ℓ (segment 4). The mass center of the system is G and is located on the axis of rotation.

Reference frames:

$R: \quad \underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ (fixed frame)

$S: \quad \underset{\sim}{i}', \underset{\sim}{j}', \underset{\sim}{k}$ (rotates with the shaft)



Find:

$\underset{\sim}{A}, \underset{\sim}{B}$ … the bearing loads at A and B

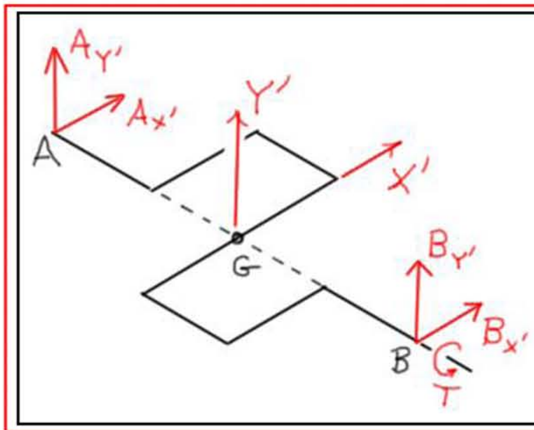
T … the driving torque

Neglect weight forces.

Solution:

Previous results for the ***angular momentum***: $\underset{\sim}{H}_G = 2m\ell^2\omega\,\underset{\sim}{i}' + (\frac{10}{3})m\ell^2\omega\,\underset{\sim}{k}$

The ***bearing loads*** at A and B can be found by applying the ***Newton/Euler equations of motion*** to the ***free-body diagram*** shown at the right.



$$\sum \underset{\sim}{F} = m\,{}^R\underset{\sim}{a}_G = \underset{\sim}{0}$$

$$\sum \underset{\sim}{M}_A = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right) + \left(\underset{\sim}{r}_{G/A} \times m\,{}^R\underset{\sim}{a}_G\right)$$

$$= \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right)$$

Terms in the moment equation:

$$\sum \underset{\sim}{M}_A = \left(\underset{\sim}{r}_{B/A} \times \underset{\sim}{B}\right) + T\,\underset{\sim}{k} = \left(-4\ell B_{Y'}\right)\underset{\sim}{i}' + \left(4\ell B_{X'}\right)\underset{\sim}{j}' + T\,\underset{\sim}{k}$$

$$\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B = 2m\ell^2\dot{\omega}\,\underset{\sim}{i}' + (\tfrac{10}{3})m\ell^2\dot{\omega}\,\underset{\sim}{k}$$

$${}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G = \omega\underset{\sim}{k} \times \underset{\sim}{H}_G = 2m\ell^2\omega^2\,\underset{\sim}{j}'$$

Substituting these results into the equations above gives the following

Force Equations

$$A_{X'} + B_{X'} = 0$$
$$A_{Y'} + B_{Y'} = 0$$

Moment Equations

$$-4\ell B_{Y'} = 2m\ell^2\dot{\omega}$$
$$4\ell B_{X'} = 2m\ell^2\omega^2$$
$$T = \tfrac{10}{3}m\ell^2\dot{\omega}$$

$\Rightarrow$

Results

$$B_{X'} = \tfrac{1}{2}m\ell\omega^2 = -A_{X'}$$
$$B_{Y'} = -\tfrac{1}{2}m\ell\dot{\omega} = -A_{Y'}$$
$$T = \tfrac{10}{3}m\ell^2\dot{\omega}$$