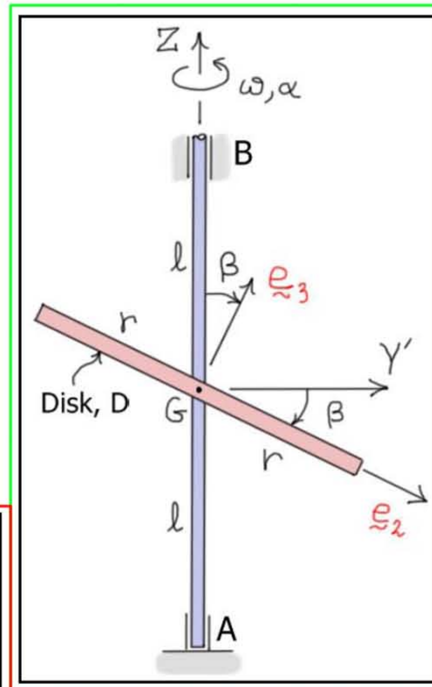
Example 15 – Intermediate Dynamics: Bearing Loads with Misaligned Disk

Reference frames: (R is the fixed frame)

$S: \; \underset{\sim}{i}', \underset{\sim}{j}', \underset{\sim}{k}$ (rotates with the shaft; aligned with the shaft)

$D: \; \underset{\sim}{i}', \underset{\sim}{e}_2, \underset{\sim}{e}_3$ (rotates with the shaft; aligned with the disk)



Find:

$\underset{\sim}{A}, \underset{\sim}{B}$ … the bearing loads at A and B

T … the driving torque

Neglect weight forces.

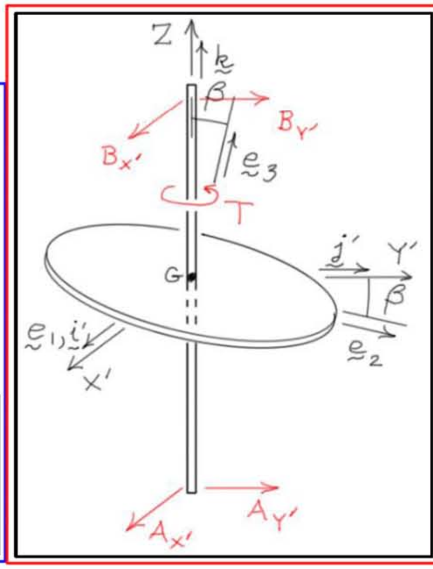
Solution:

Previous results:

$$[I_G]_D = m r^2 \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$^R\underset{\sim}{\omega}_D = \omega\, \underset{\sim}{k} = \omega\left(-S_\beta\, \underset{\sim}{e}_2 + C_\beta\, \underset{\sim}{e}_3\right)$$

$$\underset{\sim}{H}_G = \tfrac{1}{4} m r^2 \omega\left(-S_\beta\, \underset{\sim}{e}_2 + 2C_\beta\, \underset{\sim}{e}_3\right)$$
$$= \tfrac{1}{4} m r^2 \omega\left[\left(S_\beta C_\beta\right) \underset{\sim}{j}' + \left(C_\beta^2 + 1\right) \underset{\sim}{k}\right]$$



The ***bearing loads*** at A and B can be found by applying the ***Newton/Euler equations of motion*** to the ***free-body diagram***.

$$\sum \underset{\sim}{F} = \underset{\sim}{A} + \underset{\sim}{B} = m\, {}^R\underset{\sim}{a}_G = \underset{\sim}{0} \quad \Rightarrow \quad \begin{cases} A_{X'} + B_{X'} = 0 \\ A_{Y'} + B_{Y'} = 0 \end{cases} \quad \text{(scalar force equations)}$$

$$\sum \underset{\sim}{M}_A = \left(\underset{\sim}{r}_{B/A} \times \underset{\sim}{B}\right) + T\, \underset{\sim}{k} = \left(-2\ell B_{Y'}\right) \underset{\sim}{i}' + \left(2\ell B_{X'}\right) \underset{\sim}{j}' + T\, \underset{\sim}{k}$$

$$\sum \underset{\sim}{M}_A = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right) = \tfrac{1}{4} m r^2 \dot{\omega}\left[\left(S_\beta C_\beta\right) \underset{\sim}{j}' + \left(C_\beta^2 + 1\right) \underset{\sim}{k}\right] + \left(-\tfrac{1}{4} m r^2 \omega^2 S_\beta C_\beta\right) \underset{\sim}{i}'$$

Scalar moment equations:

$$\begin{gathered} -2\ell B_{Y'} = -\tfrac{1}{4} m r^2 \omega^2 S_\beta C_\beta \\ 2\ell B_{X'} = \tfrac{1}{4} m r^2 \dot{\omega}\, S_\beta C_\beta \\ T = \tfrac{1}{4} m r^2 \dot{\omega}\left(C_\beta^2 + 1\right) \end{gathered} \quad \Rightarrow \quad \begin{gathered} B_{X'} = \tfrac{1}{8\ell} m r^2 \dot{\omega}\, S_\beta C_\beta = -A_{X'} \\ B_{Y'} = \tfrac{1}{8\ell} m r^2 \omega^2 S_\beta C_\beta = -A_{Y'} \end{gathered}$$