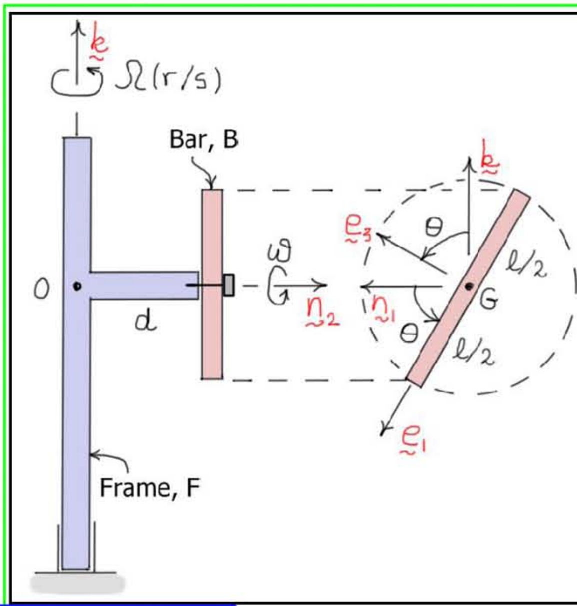
Example #16 – Intermediate Dynamics: Newton/Euler Equations of Motion

Reference frames: (R is the fixed frame)

$B:(\underset{\sim}{e}_1, \underset{\sim}{n}_2, \underset{\sim}{e}_3)$ … (rotates with the bar B)



Find:

Given Ω is ***constant***, find the ***equations of motion*** for bar B using the ***Newton/ Euler equations***.

Solution:

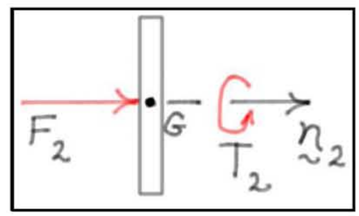
Previous results expressed in the frame B:

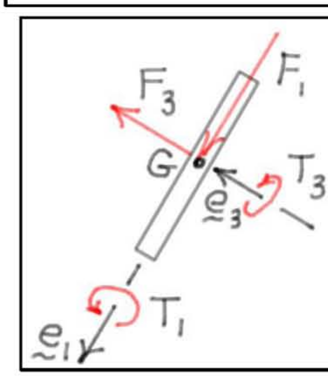
$$[I_G]_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{12}m\ell^2 & 0 \\ 0 & 0 & \frac{1}{12}m\ell^2 \end{bmatrix}$$

$${}^R\underset{\sim}{\omega}_B = (-\Omega S_\theta)\underset{\sim}{e}_1 + \omega\,\underset{\sim}{n}_2 + (\Omega C_\theta)\underset{\sim}{e}_3$$

$$\underset{\sim}{H}_G = \tfrac{1}{12} m\ell^2\left(\omega\,\underset{\sim}{n}_2 + \Omega C_\theta\,\underset{\sim}{e}_3\right)$$

The equations of motion of B can be found using the Newton/Euler equations along with the ***free-body diagrams*** shown at the right.





$$\sum \underset{\sim}{F} = F_1\,\underset{\sim}{e}_1 + F_2\,\underset{\sim}{n}_2 + F_3\,\underset{\sim}{e}_3 = m\,{}^R\underset{\sim}{a}_G = -m\,d\,\Omega^2\,\underset{\sim}{n}_2$$

$$\sum \underset{\sim}{M}_G = T_1\,\underset{\sim}{e}_1 + T_2\,\underset{\sim}{e}_2 + T_3\,\underset{\sim}{e}_3 = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right)$$

The terms on the right side of the moment equation are

$${}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G = \frac{m\ell^2}{12}\begin{vmatrix} \underset{\sim}{e}_1 & \underset{\sim}{n}_2 & \underset{\sim}{e}_3 \\ -\Omega S_\theta & \omega & \Omega C_\theta \\ 0 & \omega & \Omega C_\theta \end{vmatrix} = \frac{m\ell^2}{12}\left(\Omega^2 S_\theta C_\theta\,\underset{\sim}{n}_2 - \omega\,\Omega S_\theta\,\underset{\sim}{e}_3\right)$$

$$\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B = \frac{m\ell^2}{12}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} -\omega\,\Omega C_\theta \\ \dot{\omega} \\ -\omega\,\Omega S_\theta \end{Bmatrix} = \frac{m\ell^2}{12}\left(\dot{\omega}\,\underset{\sim}{n}_2 - \omega\,\Omega S_\theta\,\underset{\sim}{e}_3\right)$$

Inverse Dynamics
(Ω and ω are constant)

$F_1 = F_3 = 0; \quad F_2 = -md\Omega^2$

$T_1 = 0; \quad T_2 = \frac{1}{12}m\ell^2\Omega^2 S_\theta C_\theta$

$T_3 = -\frac{1}{6}m\ell^2\omega\,\Omega S_\theta$

Forward Dynamics for B
($\Omega = \text{constant}$, $\omega = \dot{\theta}$, and $\dot{\omega} = \ddot{\theta}$)

$F_1 = F_3 = 0; \quad F_2 = -md\Omega^2$

$T_1 = 0; \quad T_3 = -\frac{1}{6}m\ell^2\omega\,\Omega S_\theta$

$\ddot{\theta} + \Omega^2 S_\theta C_\theta = 12T_2/m\ell^2$

Equilibrium Positions for the Bar
$(T_2(t) \equiv 0)$

$\Omega^2 S_\theta C_\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{2}$