

## Example #17 – Intermediate Dynamics: Partial Velocities of a Slider-Crank Mechanism

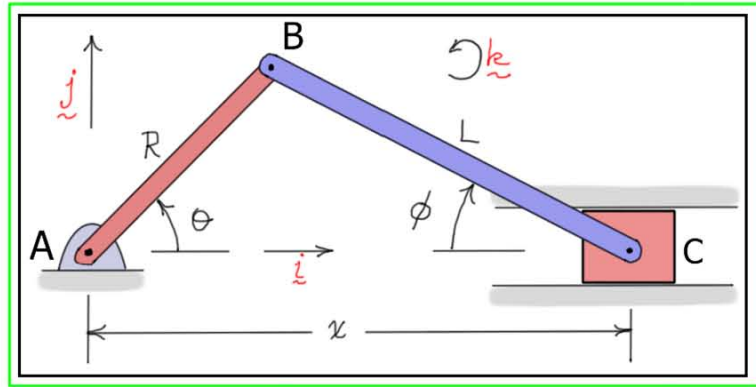
Generalized Coordinates: (not independent)

$$(q_i) = (\theta, \phi, x)$$

Find:

*Partial angular velocities* of the links

*Partial velocities* of the slider



Solution:

Constraint Equations and Differentiated Constraint Equations: (No offset)

$$\begin{cases} RS_\theta - LS_\phi = 0 \\ RC_\theta + LC_\phi - x = 0 \end{cases} \Rightarrow \begin{cases} R\dot{\theta}C_\theta - L\dot{\phi}C_\phi = 0 \\ R\dot{\theta}S_\theta + L\dot{\phi}S_\phi + \dot{x} = 0 \end{cases}$$

Angular Velocities of Links and Velocity of Piston:

$$\begin{cases} \omega_{AB} = \dot{\theta} \underline{k} = \left( \frac{LC_\phi}{RC_\theta} \right) \dot{\phi} \underline{k} \\ \omega_{BC} = -\dot{\phi} \underline{k} = -\left( \frac{RC_\theta}{LC_\phi} \right) \dot{\theta} \underline{k} \\ \underline{v}_C = \dot{x} \underline{i} = -\left( R\dot{\theta}S_\theta + L\dot{\phi}S_\phi \right) \underline{i} \end{cases}$$

Partial Angular Velocities of the Links:

$$\begin{cases} \frac{\partial \omega_{AB}}{\partial \dot{\theta}} = \underline{k} & \frac{\partial \omega_{AB}}{\partial \dot{\phi}} = \left( \frac{LC_\phi}{RC_\theta} \right) \underline{k} \\ \frac{\partial \omega_{BC}}{\partial \dot{\phi}} = -\underline{k} & \frac{\partial \omega_{BC}}{\partial \dot{\theta}} = -\left( \frac{RC_\theta}{LC_\phi} \right) \underline{k} \end{cases}$$

Partial Velocities of the Slider:

$$\begin{cases} \frac{\partial \underline{v}_C}{\partial \dot{x}} = \underline{i} \\ \frac{\partial \underline{v}_C}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}} \left[ -\left( R\dot{\theta}S_\theta + L\dot{\phi}S_\phi \left( \frac{RC_\theta}{LC_\phi} \right) \dot{\theta} \right) \underline{i} \right] = -R \left[ \frac{S_\theta C_\phi + C_\theta S_\phi}{C_\phi} \right] \underline{i} = -R \left[ \frac{S_{\theta+\phi}}{C_\phi} \right] \underline{i} \\ \frac{\partial \underline{v}_C}{\partial \dot{\phi}} = \frac{\partial}{\partial \dot{\phi}} \left[ -\left( R\dot{\theta}S_\theta + L\dot{\phi}S_\phi \right) \underline{i} \right] = -L \left[ \frac{C_\phi S_\theta + S_\phi C_\theta}{C_\theta} \right] \underline{i} = -L \left[ \frac{S_{\theta+\phi}}{C_\theta} \right] \underline{i} \end{cases}$$

Results at  $\theta = \{0, \frac{\pi}{2}, \pi\}$ :

$$\begin{cases} \frac{\partial \underline{v}_C}{\partial \dot{\theta}} = \begin{cases} \underline{0} & \text{at } \theta = 0 \\ -R\underline{i} & \text{at } \theta = \frac{\pi}{2} \end{cases} & \frac{\partial \underline{v}_C}{\partial \dot{\phi}} = \begin{cases} \underline{0} & \text{at } \theta = 0 \\ \text{undefined at } \theta = \frac{\pi}{2} \end{cases} \end{cases}$$